LT-28 Gothenburg 2017

Supercritical motion: Sami Laine
Acoustic impedance in the normal state: Juri Kuorelahti

Spin wave radiation from vortices in superfluid ^{3}He-B and supercritical motion

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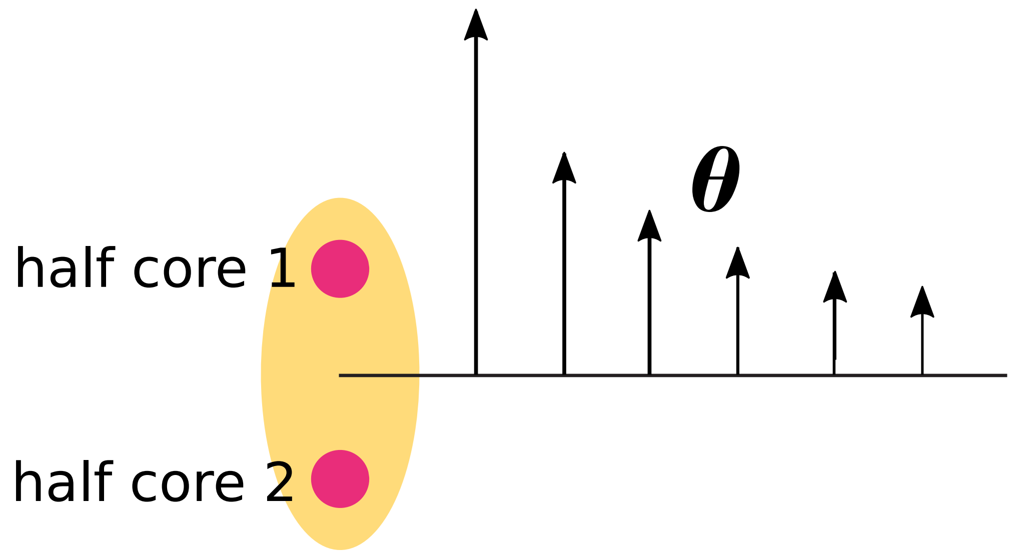
Content



Vortex structure:

- core split into two half cores
- asymptotic form far from vortex axis

Effect of precessing magnetization:

- dipole-dipole torque on the asymptotic form

⇒ dissipation by radiation of spin waves

⇒ slow rotation of the vortex

⇒ twisted vortex

- spin wave radiation from a twisted vortex

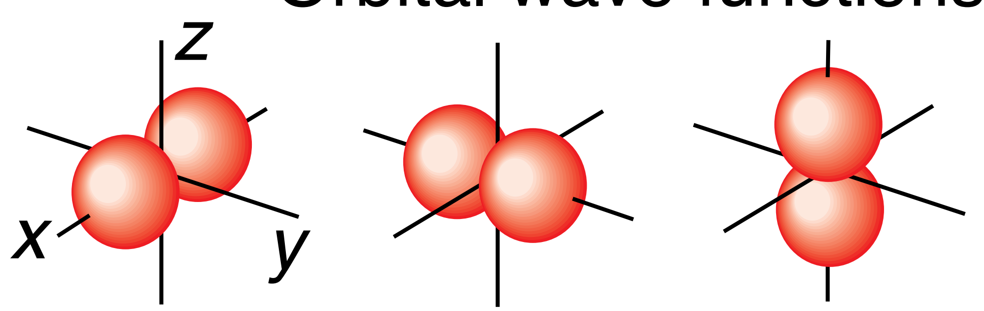
Supercritial motion

- can we understand low dissipation theoretically?

Weak coupling p-wave pairing superfluid

p–wave pairing $(L = 1)$ –> triplet pairing $(S = 1)$

Orbital wave functions (L=1)



Spin wave functions (S=1)

S_x=0: (-↑↑ + ↓↓)

S_y=0: i(↑↑ + ↓↓)

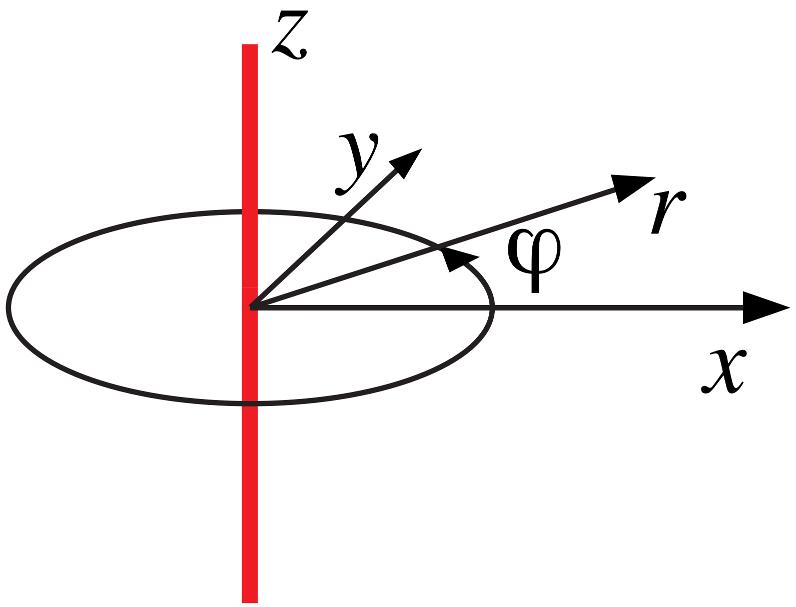
S_z=0: (↑↓ + ↓↑)

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

order parameter

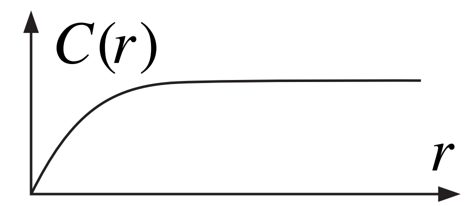
Weak coupling approximation
The ground state is Balian–Werthamer state
A = unit matrix, or arbitrary rotation matrix

Vortex



$$A_{\alpha i}(r,\varphi,z) \xrightarrow[r\to\infty]{} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} e^{i\varphi}$$

A simple solution $A_{\alpha i}(r,\varphi,z) = C(r) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} e^{i\varphi}$



Consider the x axis

far at negative x:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

smooth change

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & C(r) & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

far at positive x:

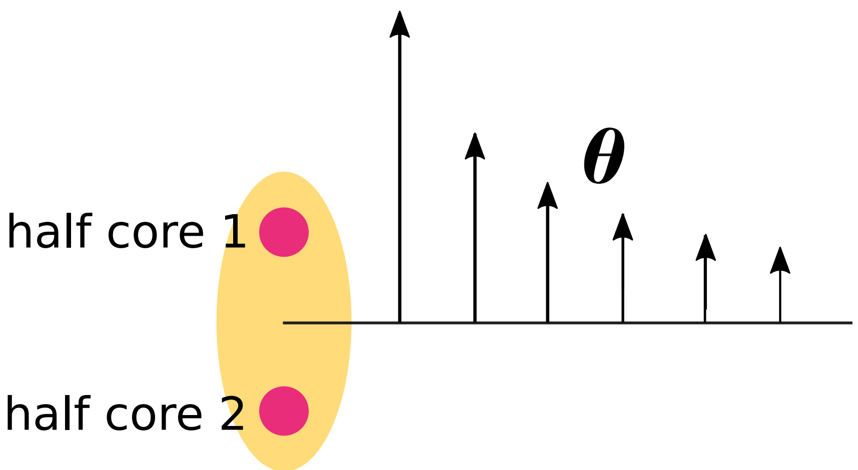
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\theta(x)$ changes from π to 0

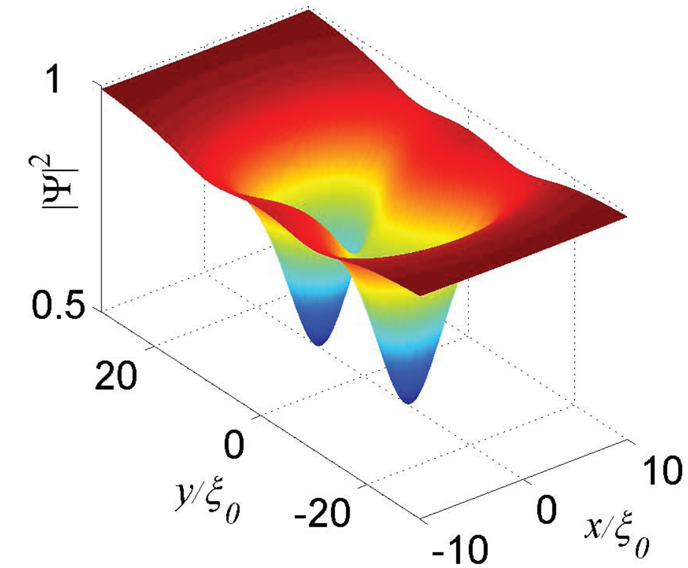
Smooth change on the x axis

pair density most suppressed at two points on the y axis:
the core is split into two half cores

Double-core vortex



Two half cores

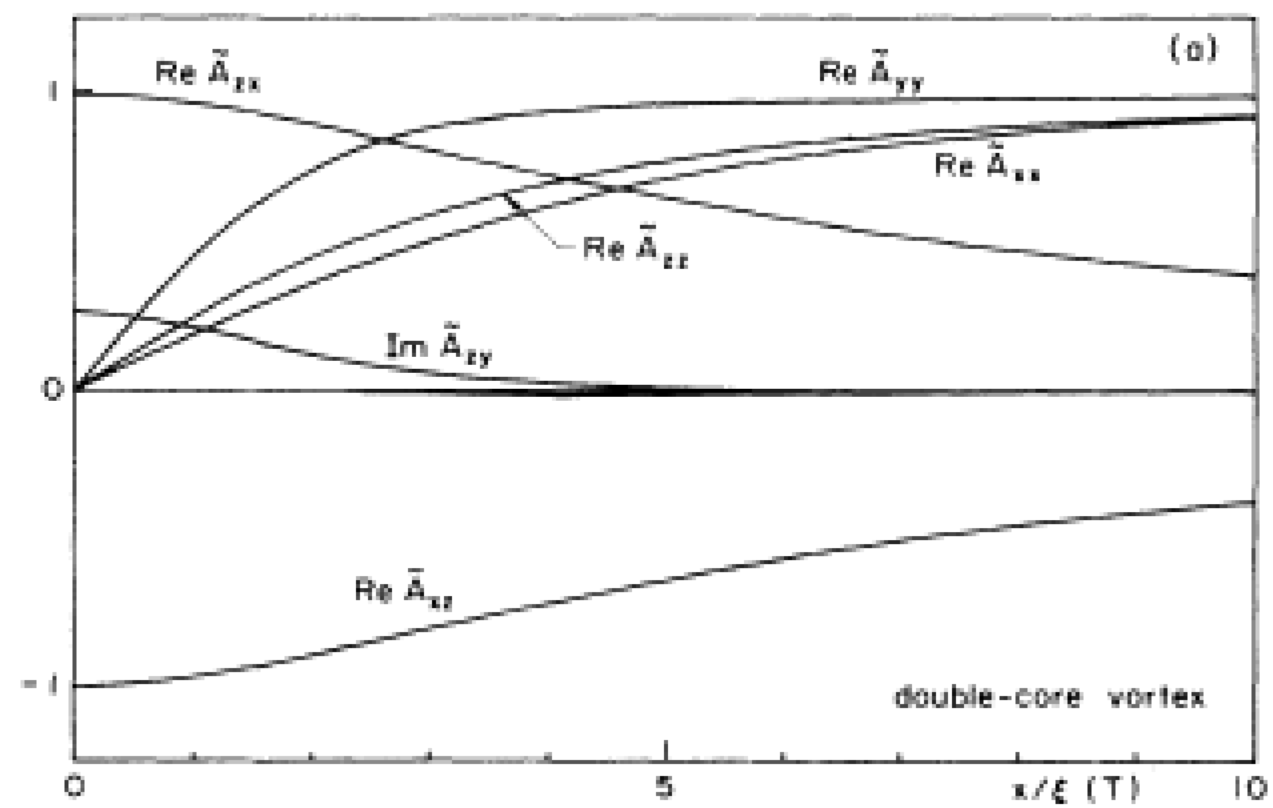


Asymptotic form far from vortex axis

$$\mathsf{A} = e^{i\varphi}\mathsf{R}(\theta_0\hat{\boldsymbol{n}})\mathsf{R}(\boldsymbol{\theta})$$

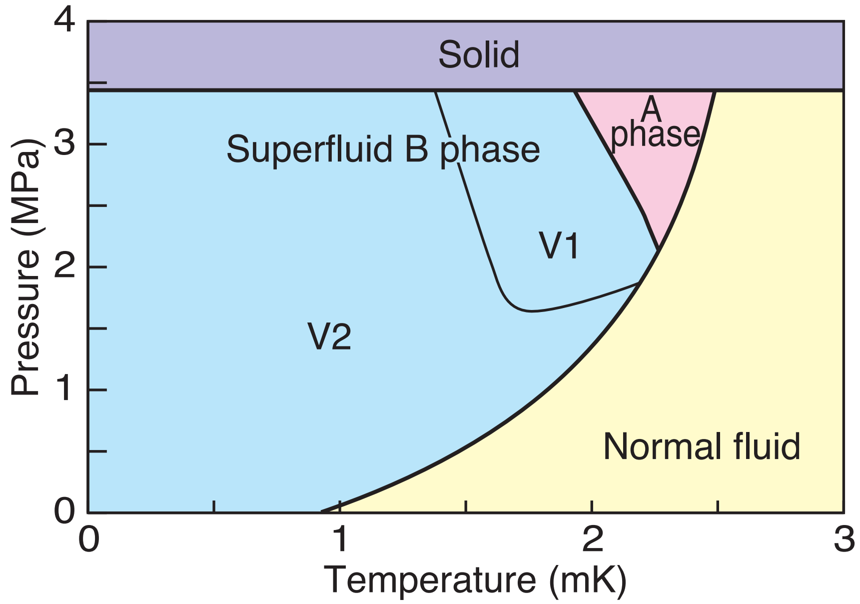
$$\boldsymbol{\theta}(r,\varphi) = \frac{C_1\cos\varphi}{r}\left(\frac{\sin\varphi}{1+c}\hat{\boldsymbol{r}} + \cos\varphi\hat{\boldsymbol{\varphi}}\right) + \frac{C_2\sin\varphi}{r}\left(-\frac{\cos\varphi}{1+c}\hat{\boldsymbol{r}} + \sin\varphi\hat{\boldsymbol{\varphi}}\right).$$

$$C_1 \gg C_2 \approx 0$$



[axially symmetric vortex: C_1=C_2 (Hasegawa 1985)]

Short history of the double-core vortex



– Experimental observation of vortex core transition in superfluid 3He-B (Ikkala, Volovik, Hakonen, Bun'kov, Islander, Kharadze, 1982) etc

– identification of the vortex structures based on calculation in the Ginzburg-Landau region (T 1986, Salomaa & Volovik 1986, T 1987): V2 = double-core vortex

– quantitative model as two half-quantum vortices bound by a domain wall (Volovik 1990)

– Broken axisymmetry was used to explain strange behavior seen in HPD mode of NMR (Kondo et al 1991)

– weak coupling calculation of the order parameter at all temperatures by self-consistent solution of Eilenberger equations (Fogelström & Kurkijärvi 1995)

– calculation of bound quasiparticle states, Lifshitz transition, rotational friction and stiffness parameters (Silaev, T & Fogelström 2015)

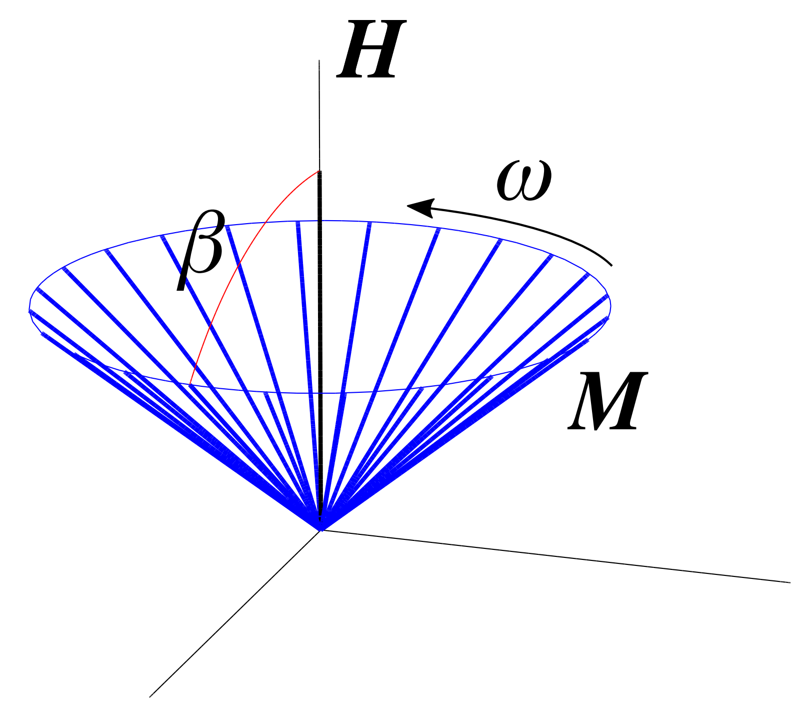
Dipole-dipole interaction

Dipole-dipole energy $$f_D = \lambda_{\rm D}(R_{ii}R_{jj} + R_{ij}R_{ji}) = \tfrac{1}{2}\lambda_{\rm D}(4\cos\vartheta + 1)^2$$

minimized with total rotation angle $$\vartheta = \vartheta_0 = \arccos(-\tfrac{1}{4}) = 104^\circ$$

– ok in the B phase equilibrium

– ok in the B phase with precessing magnetization $\boldsymbol{M}$ at tipping angles $\beta < 104^\circ$: both $\boldsymbol{M}$ and $\boldsymbol{n}$ rotate uniformly around $\boldsymbol{H}$.



– not minimized at vortices $$\vartheta = \vartheta_0 + \boldsymbol{n}\cdot\boldsymbol{\theta}$$

$$f_D = -\frac{\lambda_D}{2} + \frac{15}{2}\lambda_D\left(\boldsymbol{n}\cdot\boldsymbol{\theta}\right)^2$$
$$f_G = 2\lambda_{G2}\left[(1+c)\partial_i\theta_k\partial_i\theta_k - c\partial_i\theta_k\partial_k\theta_i\right]$$

c � 1, but $c = 0$ simplifies calculations

$$H = \frac{1}{2\chi}M^2 - \boldsymbol{\omega}_L\cdot\boldsymbol{M} + f_D + f_G$$

$\boldsymbol{M}$ and $\boldsymbol{\theta}$ conjugate variables

Spin wave radiation

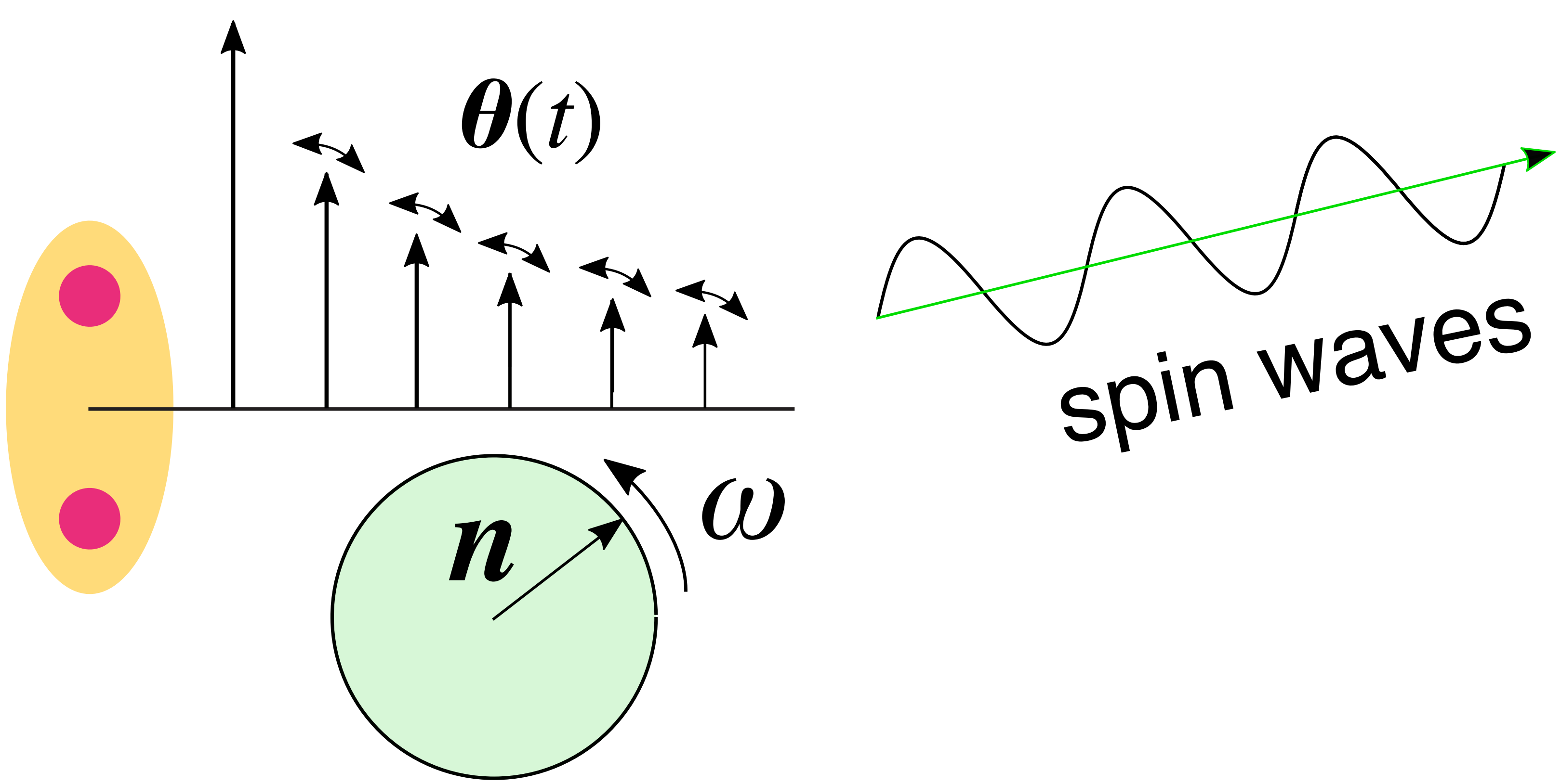
$$\ddot{\boldsymbol{\theta}} - \boldsymbol{\omega}_L \times \dot{\boldsymbol{\theta}} + \Omega^2 \boldsymbol{n}(\boldsymbol{n} \cdot \boldsymbol{\theta}) - v^2 \left[(1+c)\nabla^2 \boldsymbol{\theta} - c\nabla\left(\nabla \cdot \boldsymbol{\theta}\right)\right] = 0$$

$$\boldsymbol{\theta}(\boldsymbol{r}, t) = \boldsymbol{\theta}_1(\boldsymbol{r}) + \boldsymbol{\theta}_2(\boldsymbol{r}, t)$$

$\Rightarrow$ inhomogeneous wave equation

$$\ddot{\boldsymbol{\theta}}_2 - \boldsymbol{\omega}_L \times \dot{\boldsymbol{\theta}}_2 + \Omega^2 \boldsymbol{n}\boldsymbol{n} \cdot \boldsymbol{\theta}_2 - v^2 \left[(1+c)\nabla^2 \boldsymbol{\theta}_2 - c\nabla\left(\nabla \cdot \boldsymbol{\theta}_2\right)\right] = -\Omega^2 \boldsymbol{n}\boldsymbol{n} \cdot \boldsymbol{\theta}_1$$

$\Rightarrow$ eigenvalue equation for three polarizations: one decaying, two propagating modes

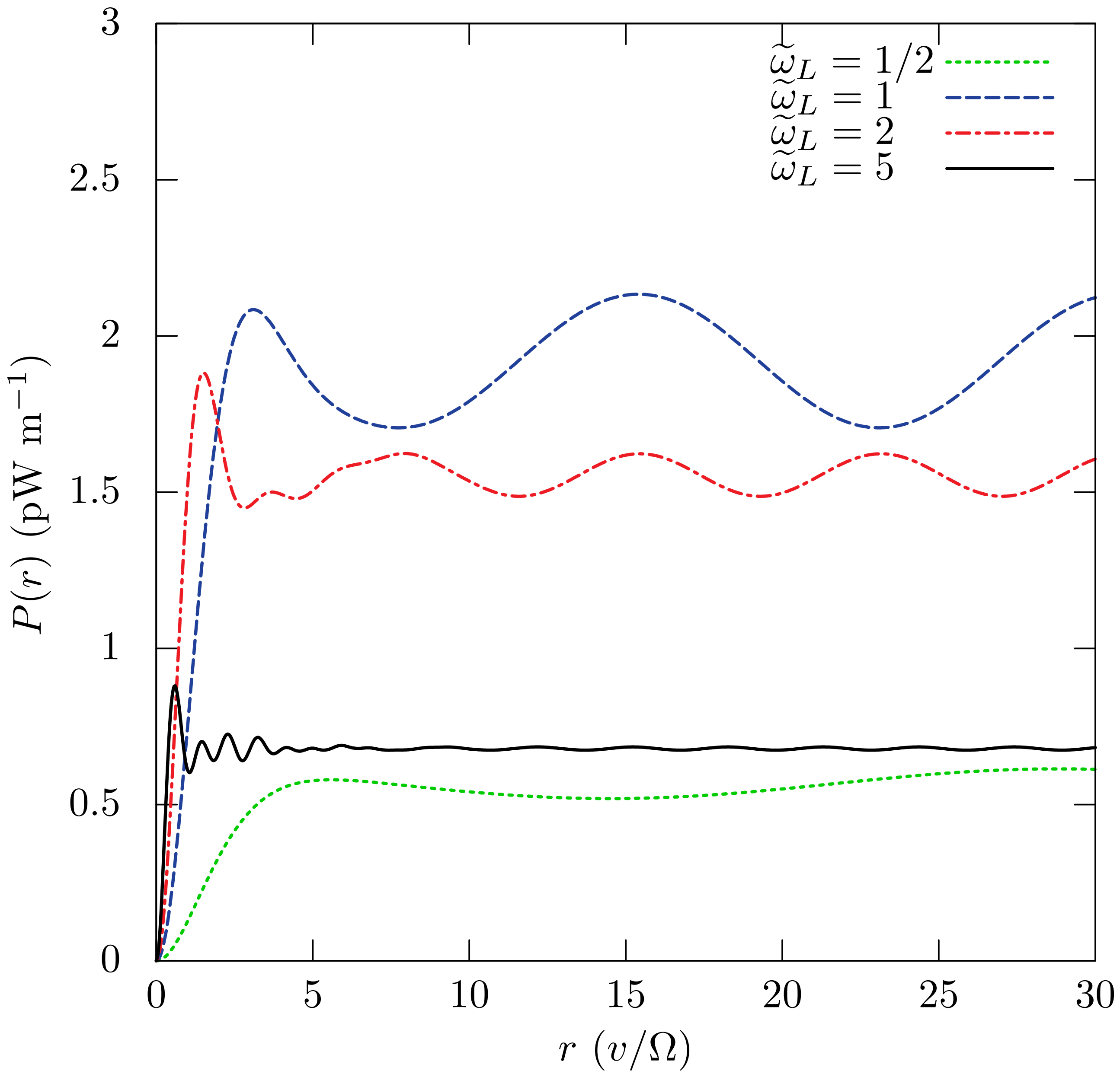


Radiated power

energy flux

$$\Sigma_i = -\chi v^2 \left[(1+c)\dot{\theta}_k \partial_i \theta_k - c\dot{\theta}_k \partial_k \theta_i \right]$$

radiated power



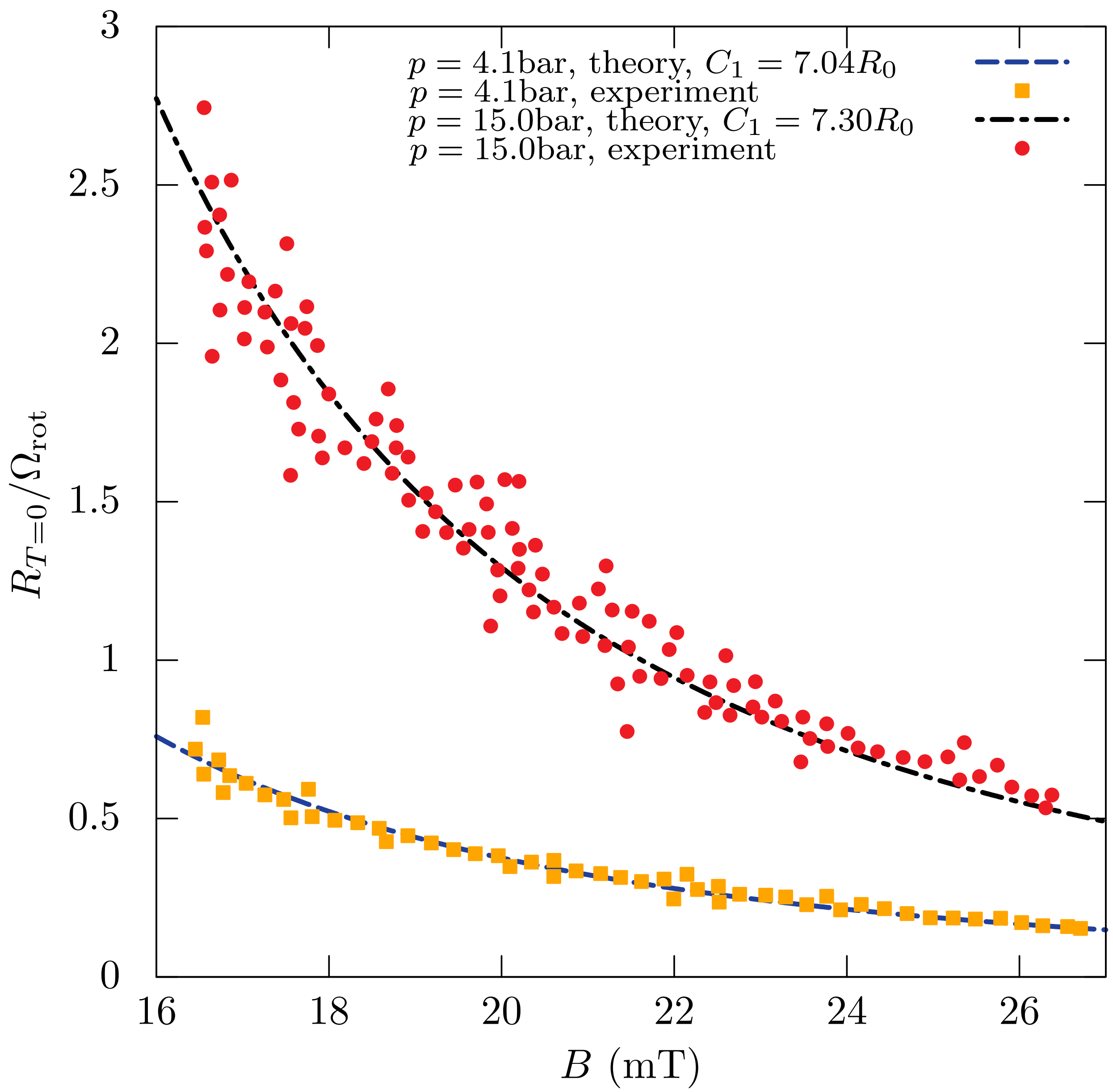
radius

Results for small tipping, small T/T_c

$$P = \frac{\pi^2}{8}\chi\Omega^4 \frac{\omega_L}{\omega_L^2 - \Omega^2}\frac{4\beta^2}{5}\left\{\frac{3c^2+6c+4}{4(1+c)^2}\left(C_1^2+C_2^2\right)+\frac{2c(2+c)}{4(1+c)^2}C_1C_2\right\}H\left(\omega_L-\Omega\right)$$

step function

Relaxation rate vs. magnetic field

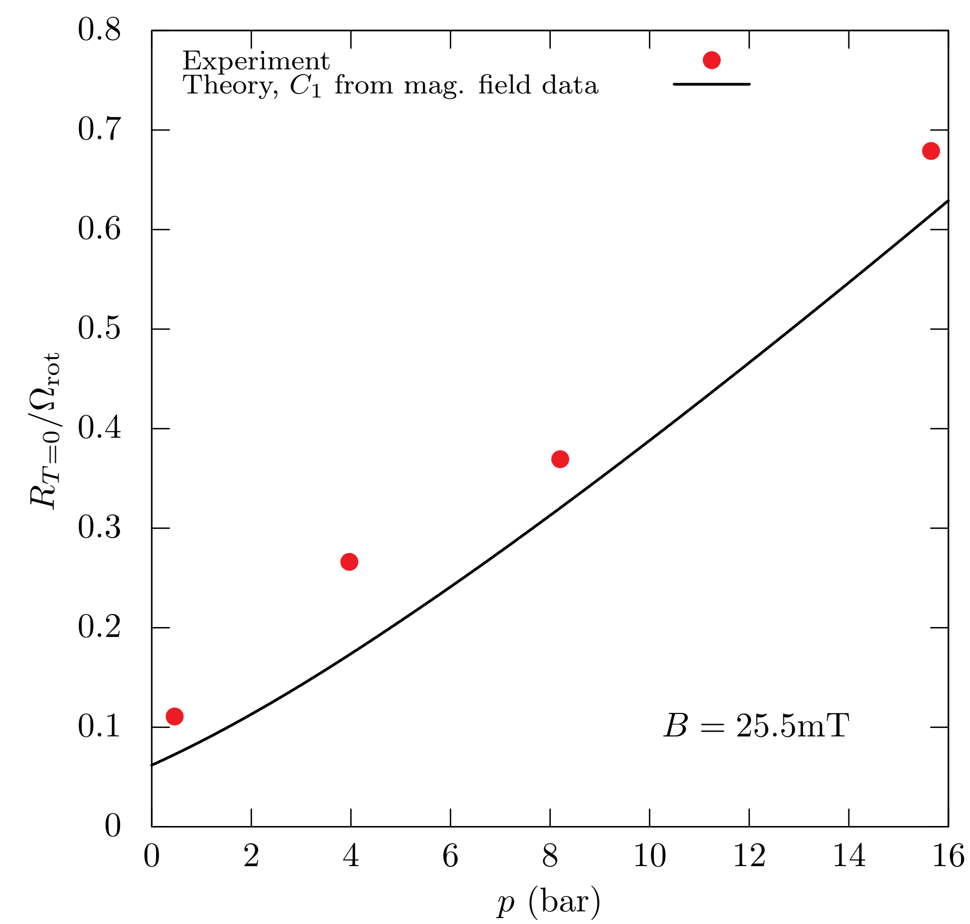


Experiment: Rota group (Aalto)

Only fitting parameter $C_1 \approx 7$ ($C_2 = 0$)

Weak coupling calculation of the vortex structure: $C_1 \approx 5$
(Silaev, T, Fogelström 2015)

pressure dependence

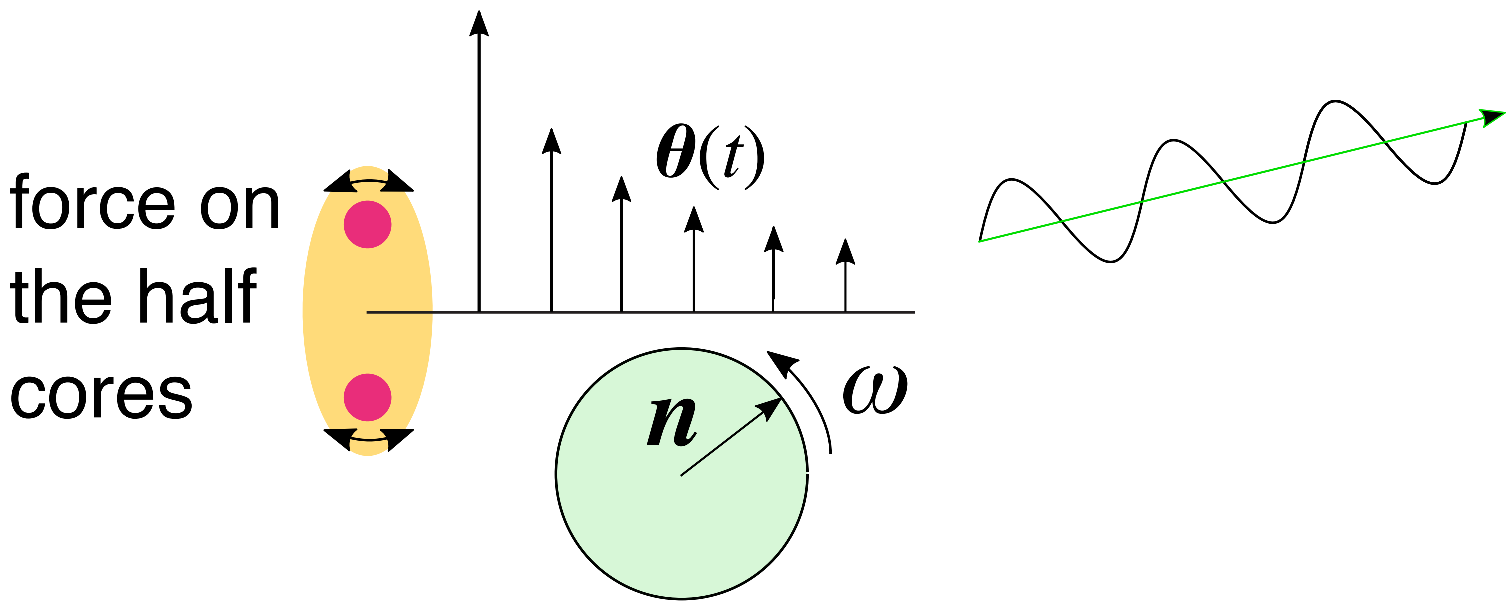


Other relaxation mechanisms

1) nonequilibrium between normal and superfluid components (Leggett & Takagi 1977)

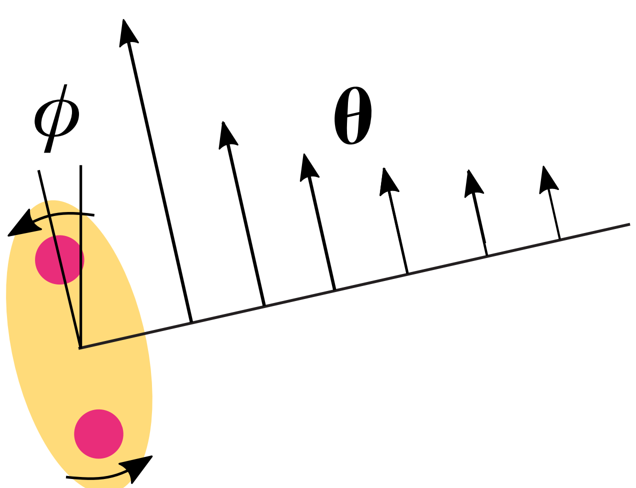
– weak at low temperatures, contributes at high temperatures (Laine & T 2016)

2) dissipation by bound quasiparticle states in the half cores (Kondo et al 1991).



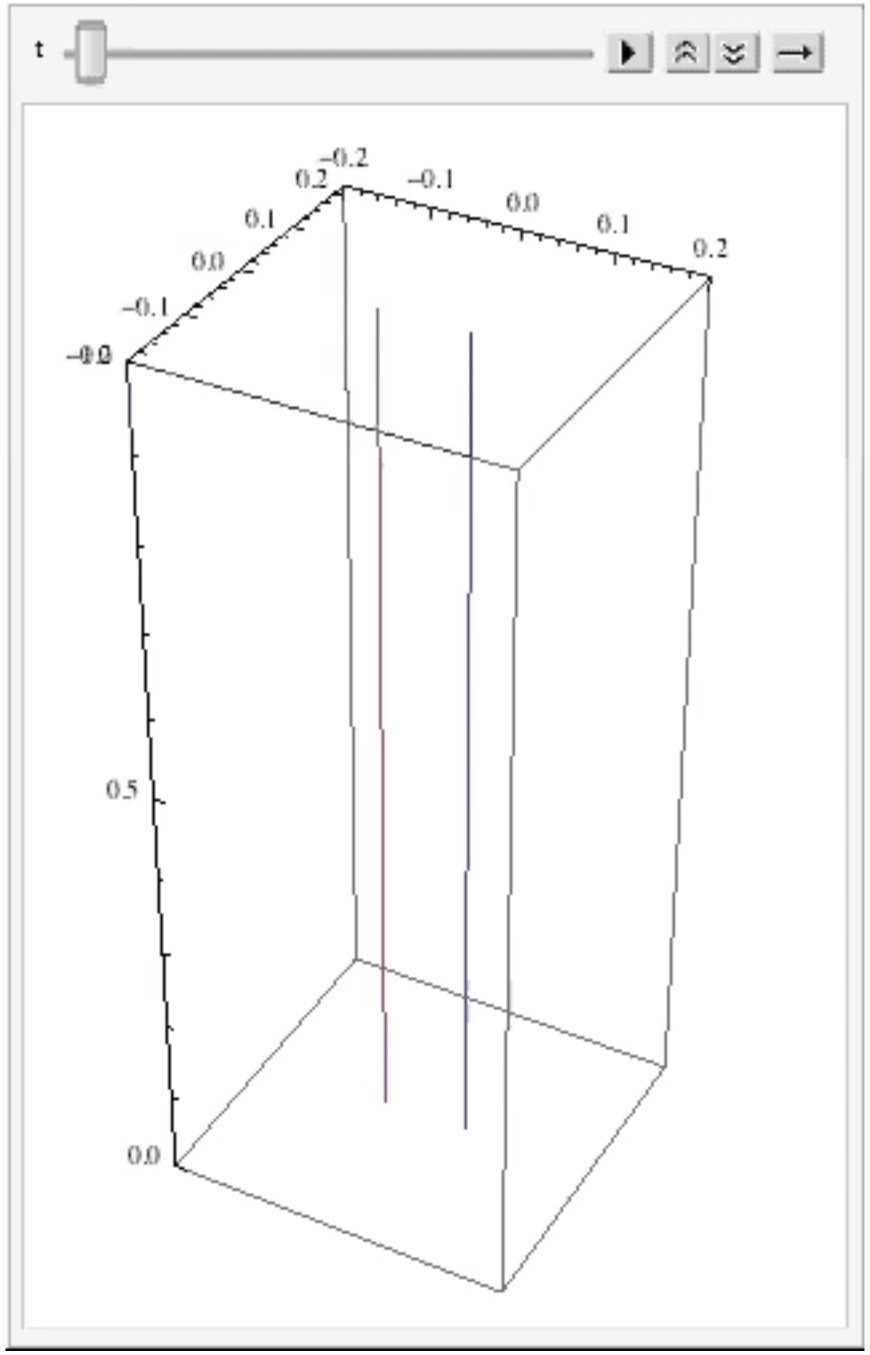
– calculation of the bound states (Silaev, T & Fogelström): the friction coefficient f is so large that the dissipation is negligible

– softness of $\boldsymbol{\theta} \Rightarrow$ d.c. force $\Rightarrow$ slow rotation of the vortex



$$\dot{\phi} = \frac{P}{\omega f} + \frac{K}{f}\frac{\partial^2 \phi}{\partial z^2}$$

Twisted double-core vortex



Suppose the vortex is pinned at the top and bottom walls (cell height L)

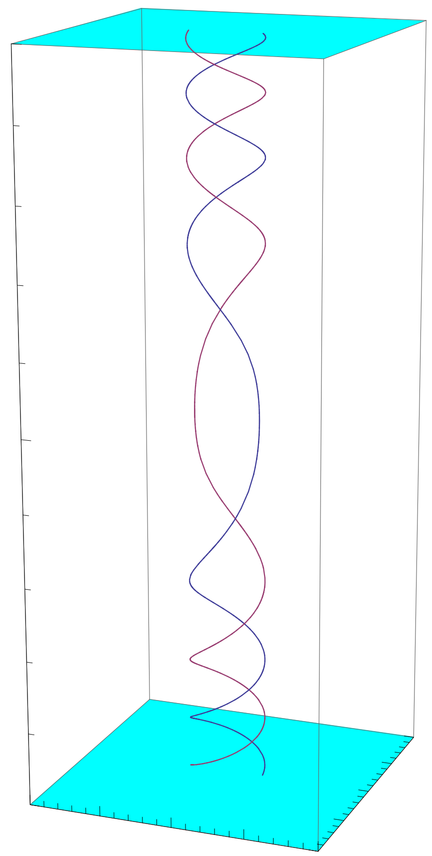
Twisting of the double-core vortex was suggested by Kondo, Korhonen, Krusius, Dmitriev, Mukharsky, Sonin, and Volovik (1991)

We can now understand the time scale: the diffusion equation

$$\dot{\phi} = \frac{P}{\omega f} + \frac{K}{f}\frac{\partial^2 \phi}{\partial z^2}$$

and the calculation of K and f give the time scale $\dfrac{L^2 f}{\pi^2 K}$ of a few minutes

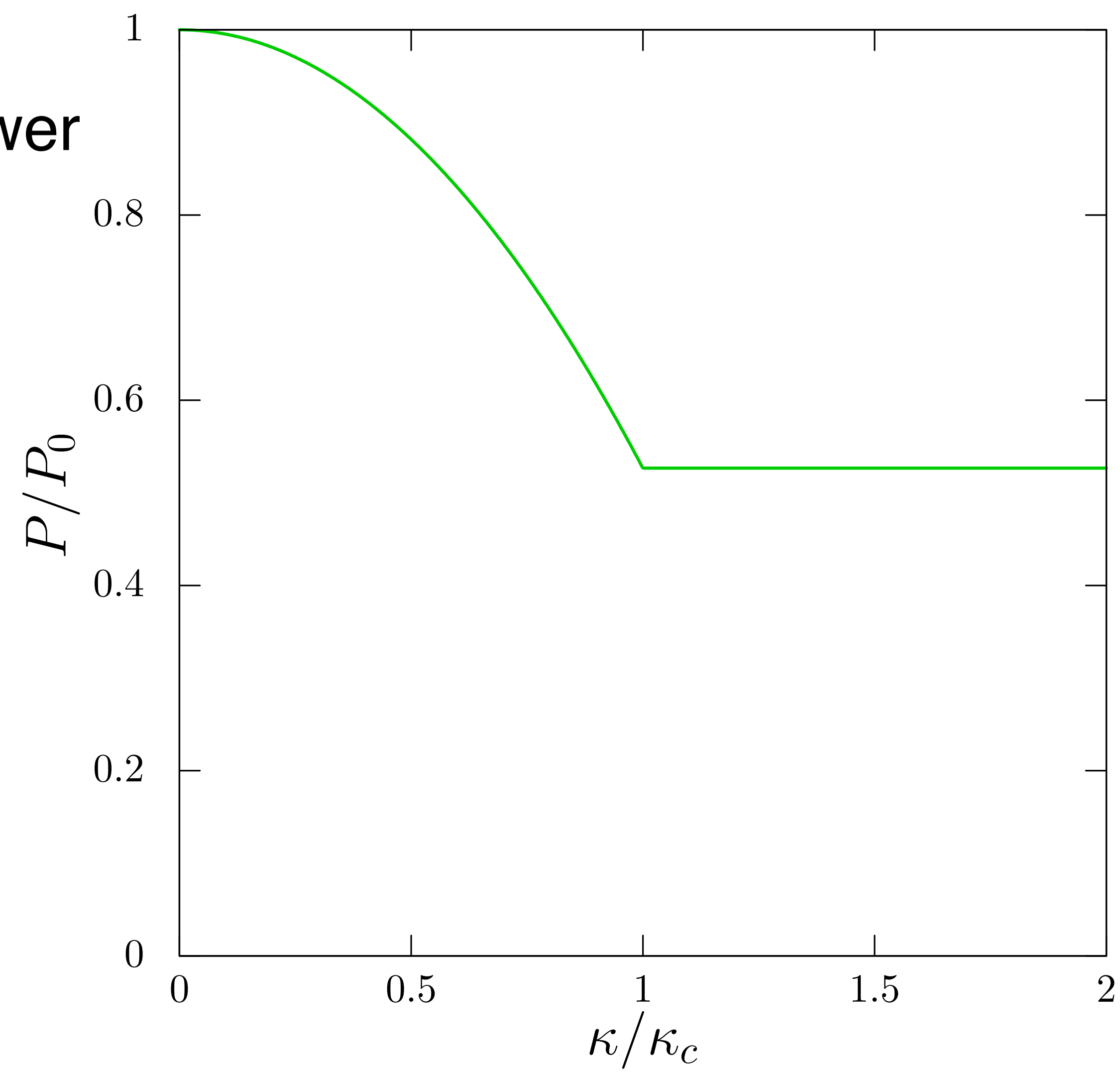
Spin-wave radiation from a twisted vortex



$$\ddot{\boldsymbol{\theta}} - \boldsymbol{\omega}_L \times \dot{\boldsymbol{\theta}} + \Omega^2 \boldsymbol{n}(\boldsymbol{n} \cdot \boldsymbol{\theta}) - v^2 \left[(1+c)\nabla^2 \boldsymbol{\theta} - c\nabla\left(\nabla \cdot \boldsymbol{\theta}\right)\right] = 0$$

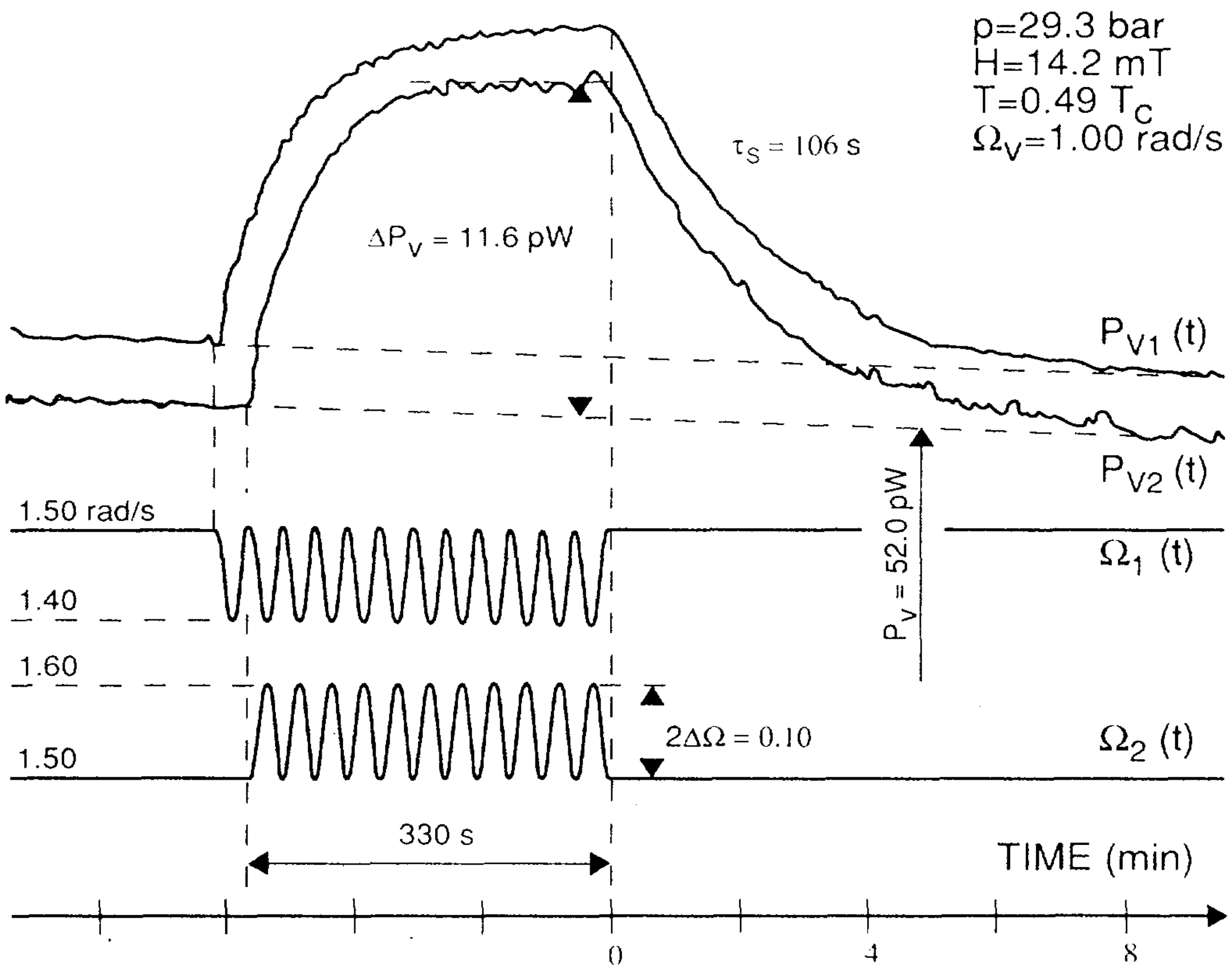
Twisting reduces the coherence of the emitted waves
One of the two modes becomes nonpropagating at a critical twisting

radiated power



twisting

Experimental data on twisted vortices



Krusius, Kondo, Korhonen, Sonin, Phys. Rev. B 47, 15113 (1993)

theory explains

- changes of absorption
- time scales (including the factor of 2 difference in increasing and decreasing absorption)

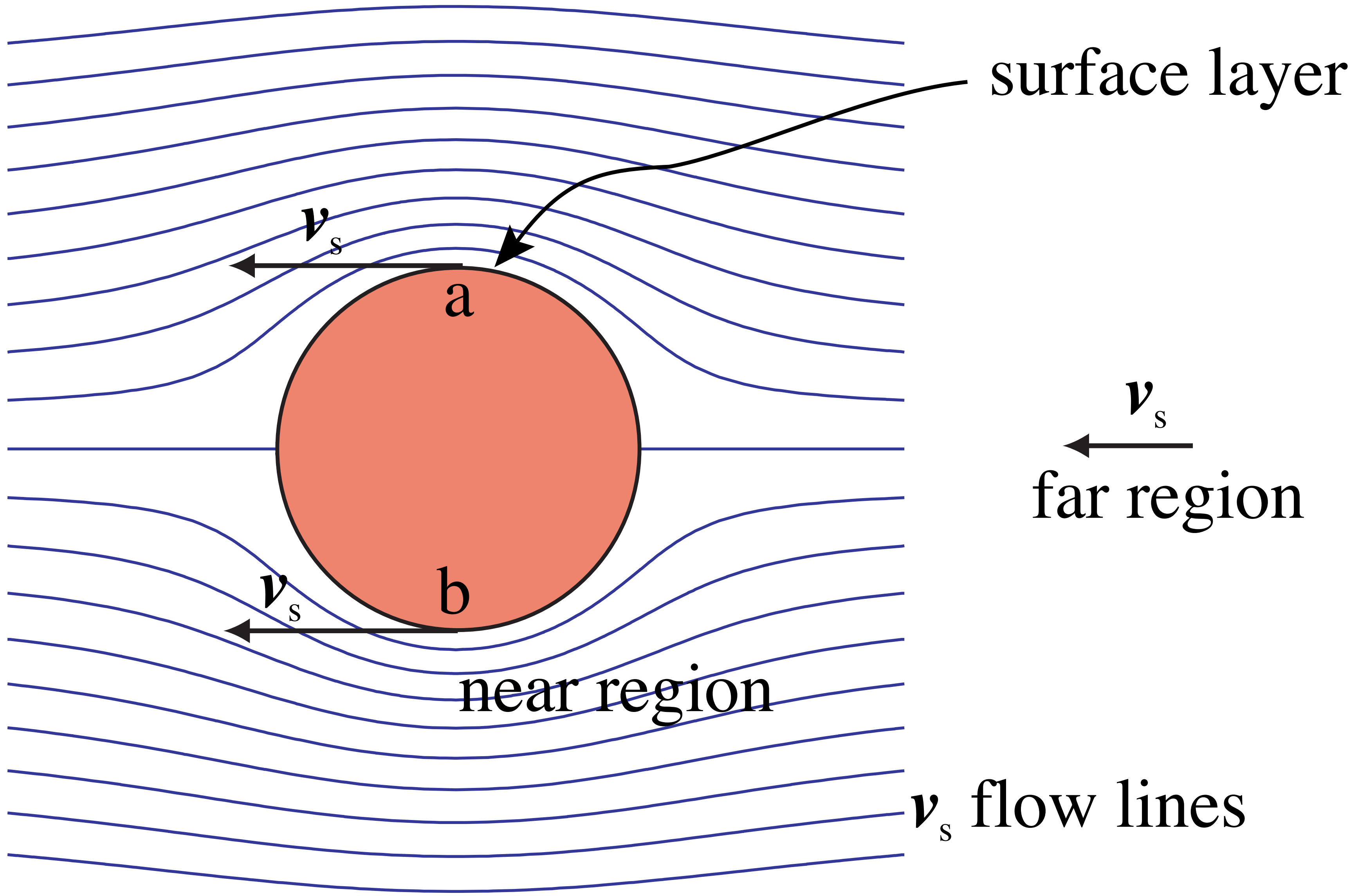
Summary of spin wave radiation in vortices of ^{3}He-B

Spin wave radiation is the dominant dissipation mechanism of NMR in vortices at temperatures $T/T_c < 0.5$

Theory and experiment agree essentially without fitting parameters

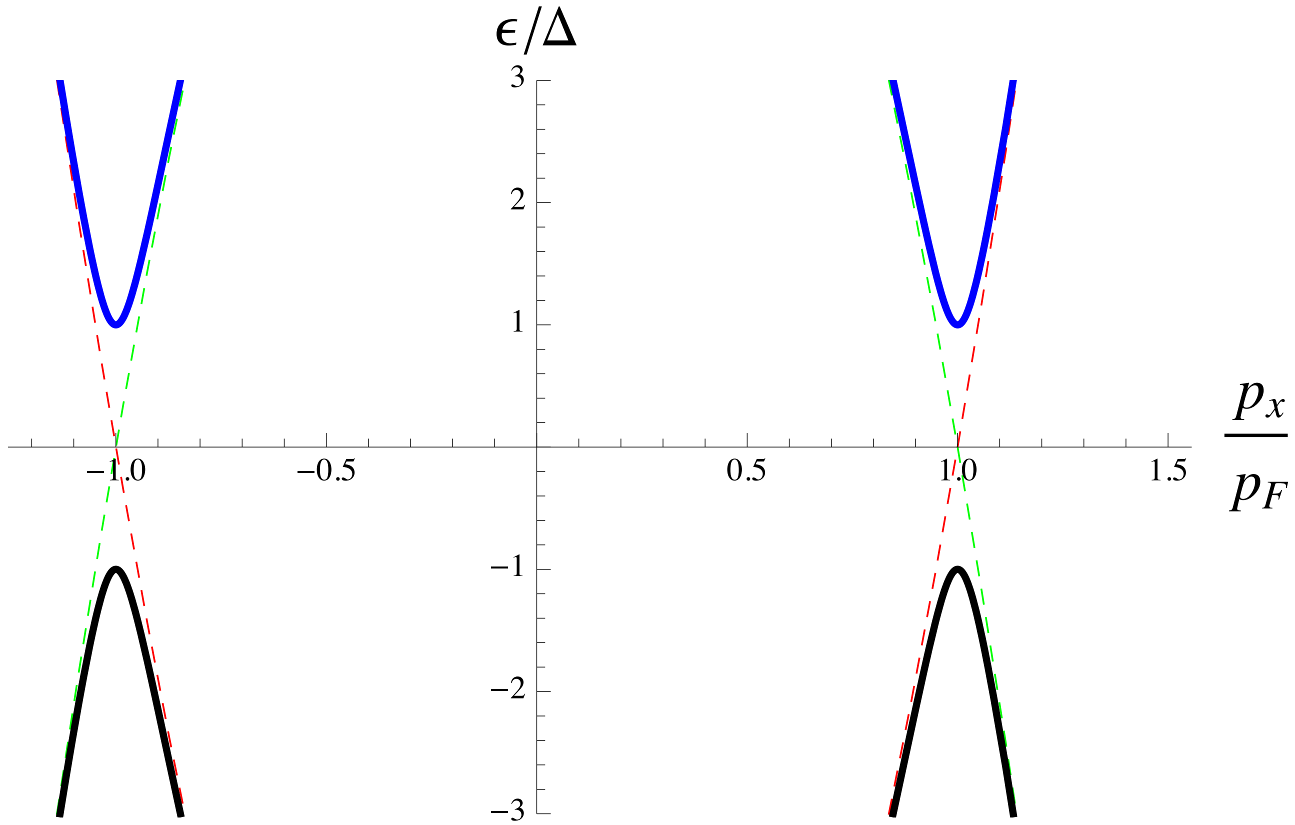
Poster 780 by Sami Laine on Monday

Moving objects in a Fermi superfuid: three regions

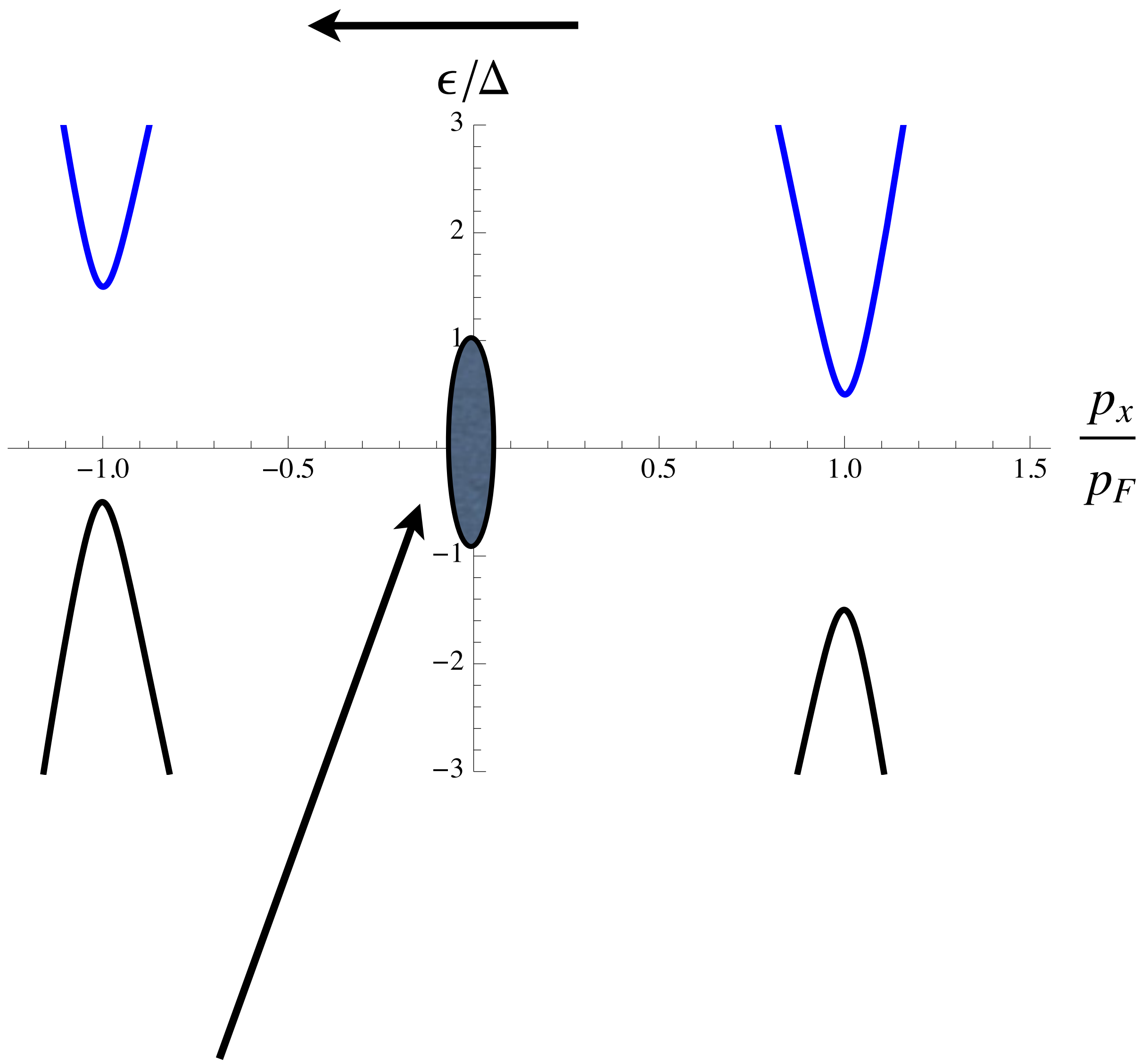


Quasiparticle dispersion

stationary superfluid



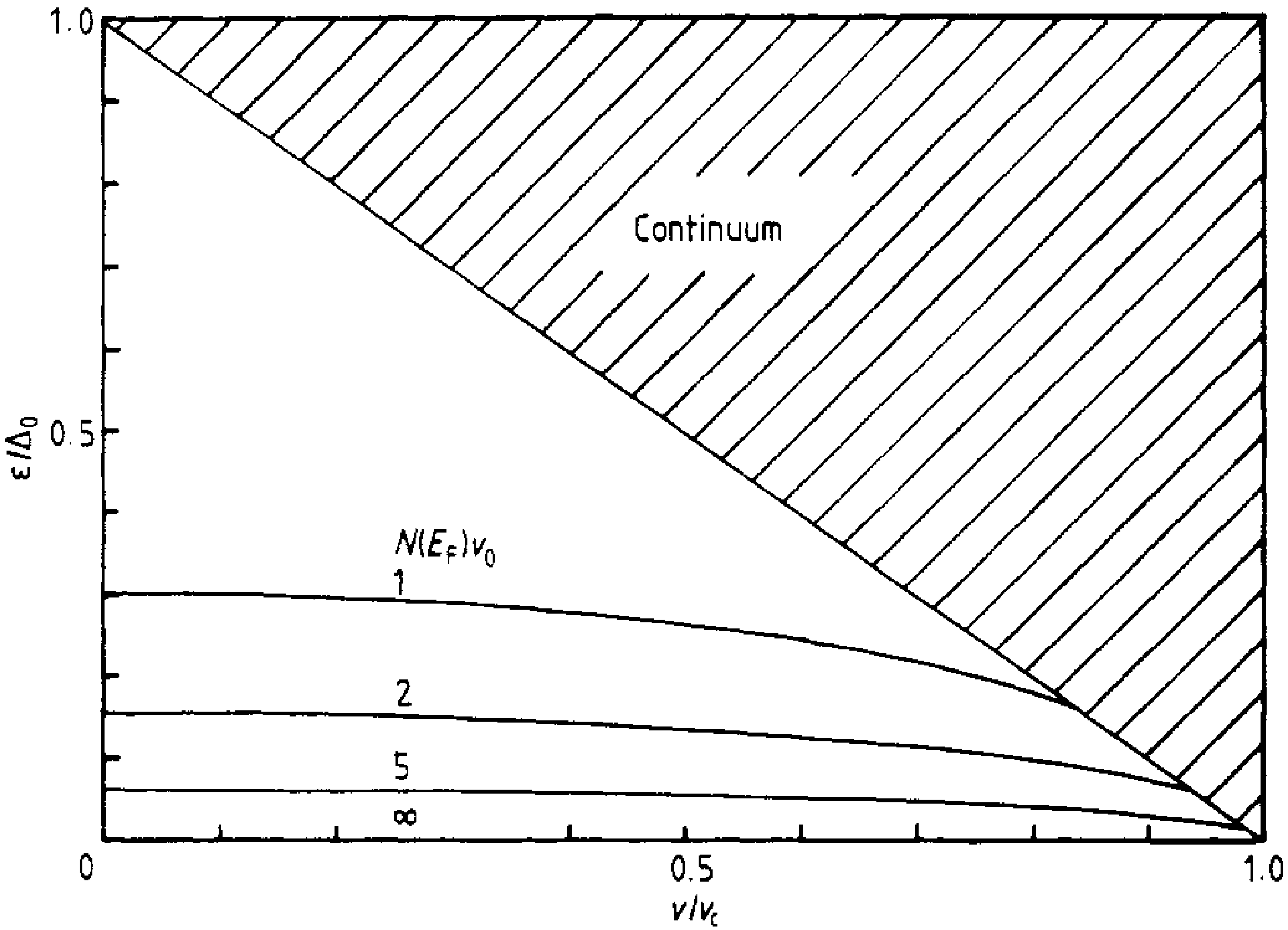
moving superfluid



bound excitations in the surface layer for diffuse scattering

How can Lambert mechanism work?

Bound state energies for a point object



v/v_L

Ashauer, Rainer 1988

No crossing of the Fermi level at subcritical velocity

The same seems to hold for a wall

Superfluid low frequency dynamics

Serene & Rainer 1983

the shift of quasiparticle energies is determined selfconsistently by the excitations

$$a = m v_F \boldsymbol{v}_s \cdot \hat{\boldsymbol{p}} + \frac{1}{2} \frac{F_1^s}{1 + \frac{1}{3} F_1^s} \int \frac{d\Omega_p'}{4\pi} \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{p}}' \int_{-E_c}^{E_c} d\epsilon \, N (\phi_{B1} + \phi_{B2})$$

Two limiting cases

1) short time scale: ignore collisions between quasiparticles

⇒ It seems that a supercritical state can be stabilized in the near region

- it seems unlikely that such a low dissipation can be reached as reported by Bradley et al (2016)

What happens between these limits?

2) long time scale: equilibrium is achieved in the near region through collisions between quasiparticles

⇒ vortex formation