LT-26, Beijing, Aug. 15, 2011

Pendulum in a Fermi liquid

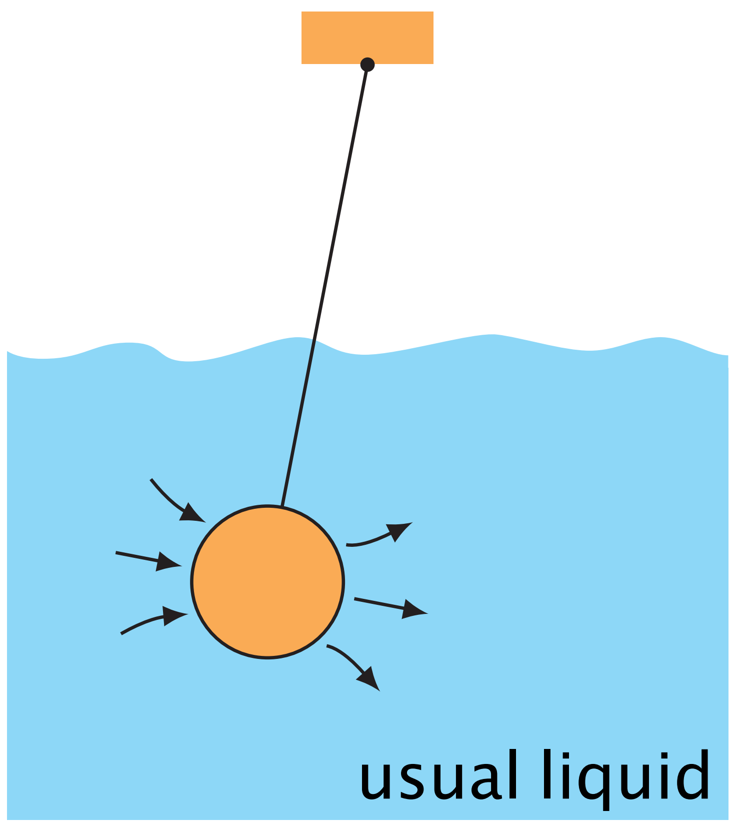
Erkki Thuneberg
Timo Virtanen

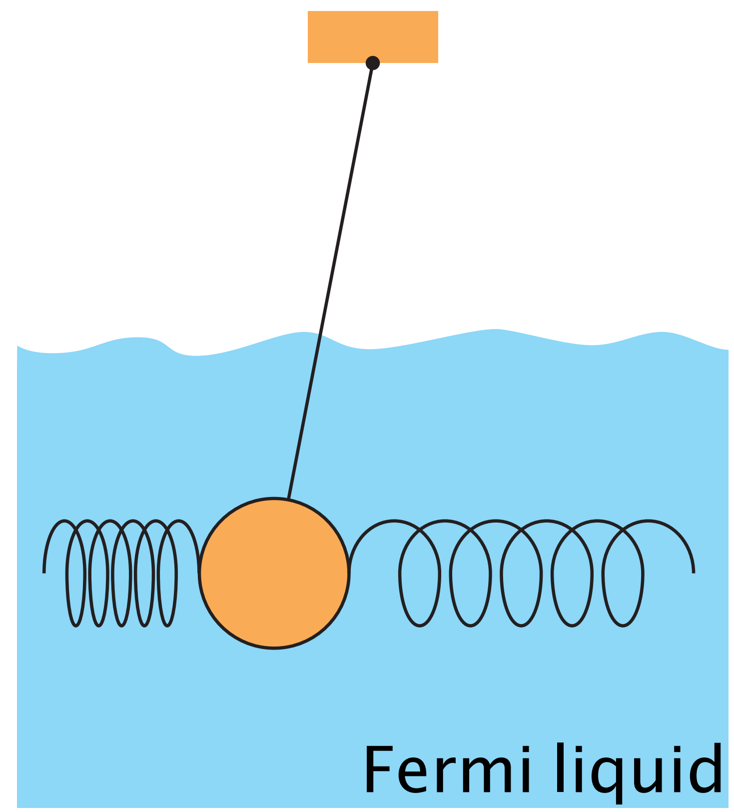
University of Oulu
Finland

Purpose

- to explain vibrating wire measurements in ^{3}He–^{4}He mixtures
- to understand Fermi liquid theory

Immerse a pendulum in a liquid





The liquid is dragged into motion. The increased effective mass reduces the oscillation frequency.

The liquid can act like an elastic medium, increasing the oscillation frequency.

Content

brief introduction to Fermi liquid theory

explanation of the Landau force

Fermi-liquid equations for ^{3}He-^{4}He mixtures

calculation of the response to an oscillating cylinder

comparison to experiments

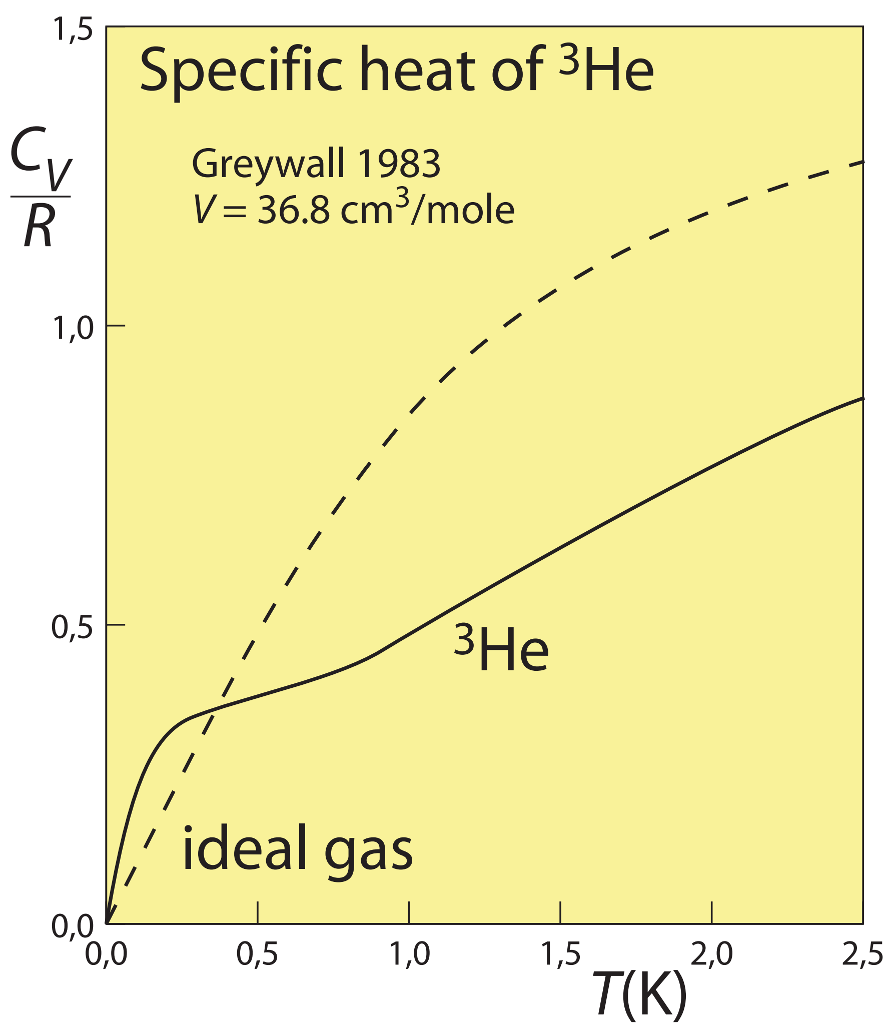
further consequences

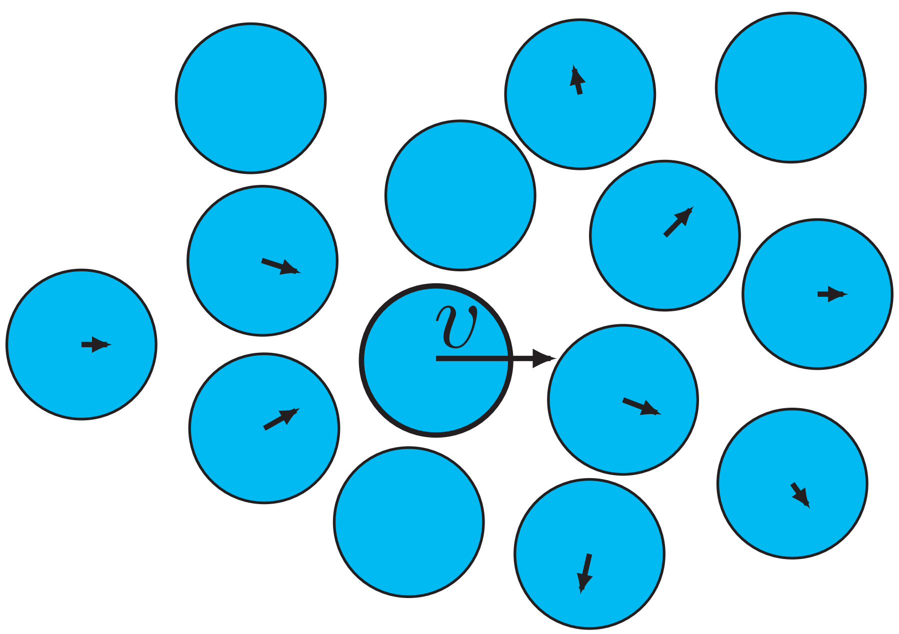
Fermi liquid theory

The problem of many interacting particles

Lev Landau developed a model how an interacting Fermi-system can behave (1957)

Instead of strongly interacting particles, there are quasiparticles, that interact only weakly





$$\boldsymbol{p} = (m_F + \delta m)\boldsymbol{v}$$

excitation velocity

additional mass

fermion mass

excitation momentum

Fermi liquid interactions

Why interactions?

Suppose giving all particles an extra velocity $\boldsymbol{v}$

$$\boldsymbol{P} = N(m_F + \delta m)\boldsymbol{v} \neq N m_F \boldsymbol{v}$$

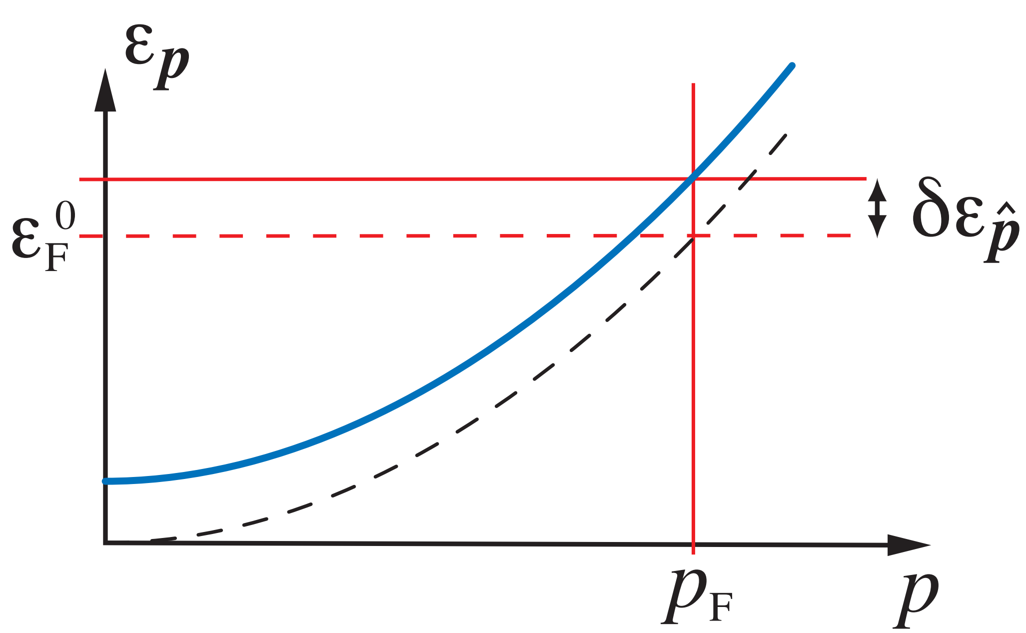
total momentum

number of particles

$$\epsilon_{\boldsymbol{p}} = \frac{p^2}{2(m_F + \delta m)} + \delta\epsilon_{\hat{\boldsymbol{p}}}$$

$$\delta\epsilon_{\hat{\boldsymbol{p}}} = \int d^3p' f(\boldsymbol{p}, \boldsymbol{p}')(n_{\boldsymbol{p}} - n_p^{(0)})$$

quasiparticle distribution function



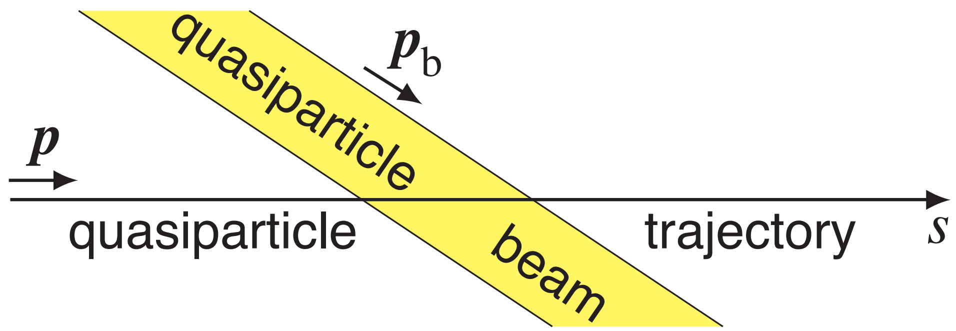
The interaction $\delta\varepsilon$ shifts the energies just to compensate δm when the whole Fermi sphere is shifted

The interaction is parametrized by

$$F_0^s,\ F_1^s,\ F_2^s, \ldots,\ F_0^a,\ F_1^a,\ F_2^a, \ldots$$

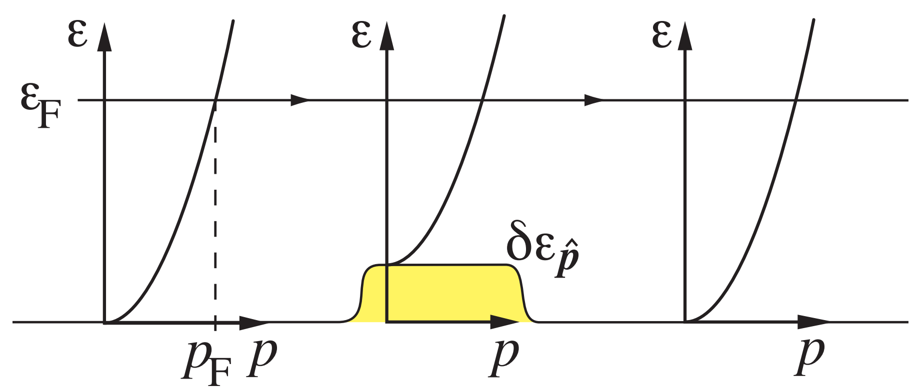
Effect of Fermi liquid interactions

Assume a beam of quasiparticles



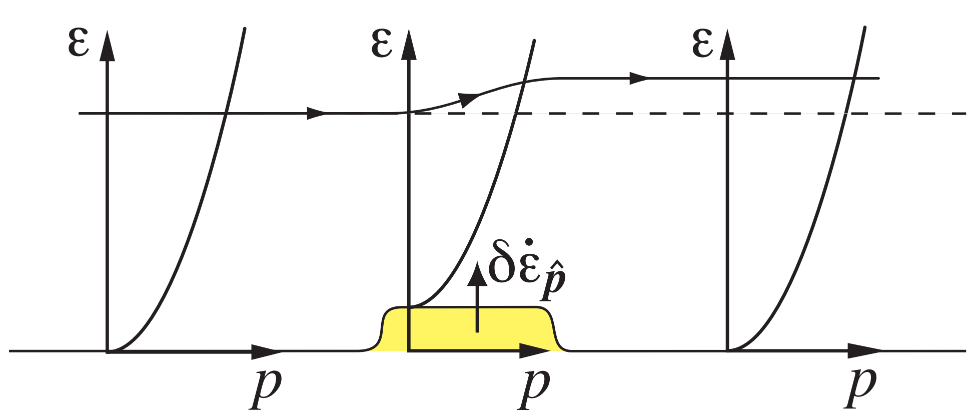
study a quasiparticle trajectory crossing the beam

the beam causes a potential on the crossing trajectory



a stationary quasiparticle beam

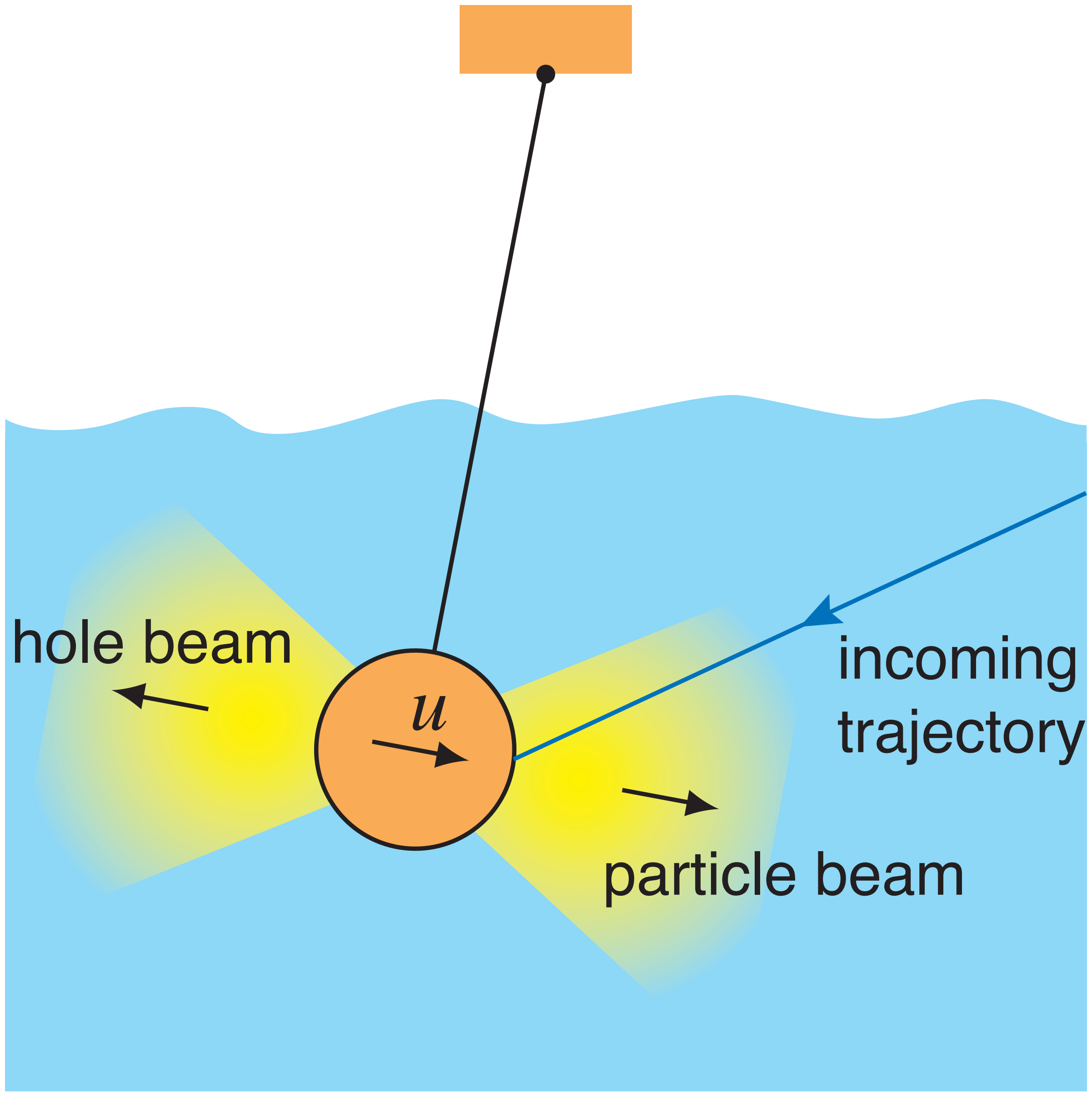
⇒ negligible effect at the Fermi level



time-dependent quasiparticle beam

⇒ radiation/absorption of quasiparticles

Elasticity/inertia of Fermi liquid



a moving object generates a beam of quasiparticles

the beam causes a potential on incoming quasiparticles

assume F_0 most important interaction

$\delta\dot{\epsilon} \propto F_0 \dot{u}$

a) pure ^{3}He: $F_0 > 0$
⇒ more particles hit the pendulum in front
⇒ increased inertia

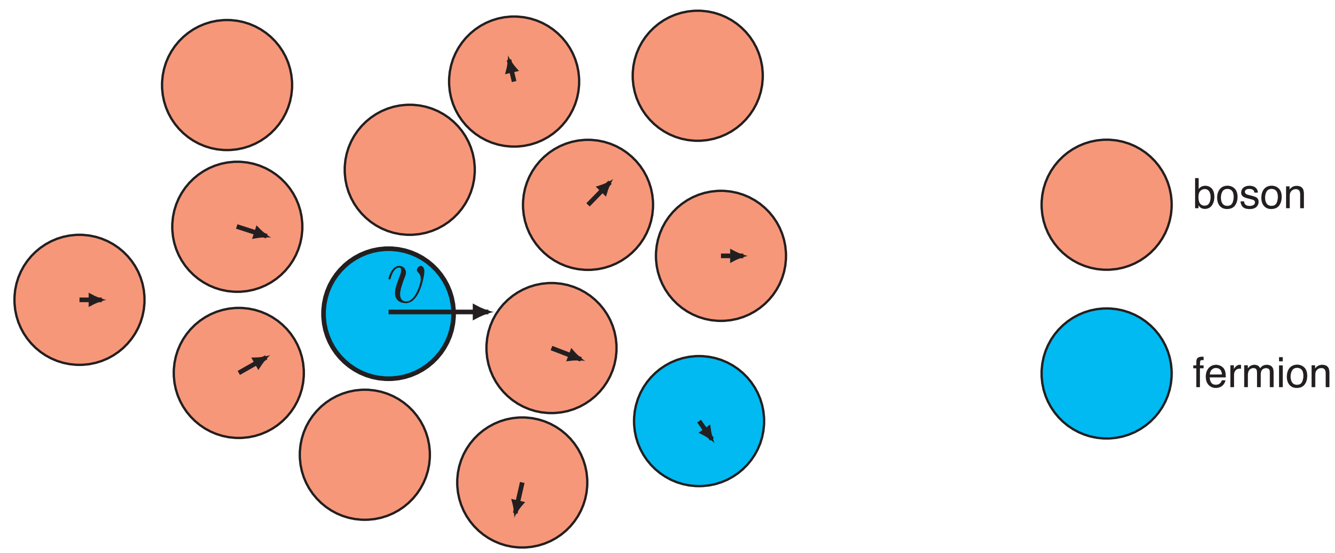
b) ^{3}He–^{4}He mixture : $F_0 < 0$
⇒ less particles hit the pendulum in front
⇒ elasticity of the liquid

“Landau force”

Fermi-liquid theory for fermion-boson mixture

(^3He–^4He mixture)

Khalatnikov (1969)



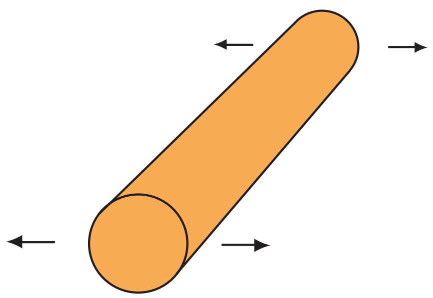
quasiparticle momentum

$$\boldsymbol{p} = (m_F + \delta m_B + \delta m_F)\boldsymbol{v}$$

$$\delta\epsilon_{\hat{\boldsymbol{p}}}(\boldsymbol{r},t) = (1+\alpha)\delta\mu_{\rm B}(\boldsymbol{r},t) + Dp_{\rm F}\hat{\boldsymbol{p}}\cdot\boldsymbol{v}_s(\boldsymbol{r},t) + \sum_{l=0}^{\infty} F_l \langle P_l(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}')\phi_{\hat{\boldsymbol{p}}'}(\boldsymbol{r},t)\rangle_{\hat{\boldsymbol{p}}'},$$

$$\phi_{\hat{\boldsymbol{p}}} = \int [n_{p\hat{\boldsymbol{p}}} - n_p^{(0)}] v_{\rm F} dp$$

Calculation



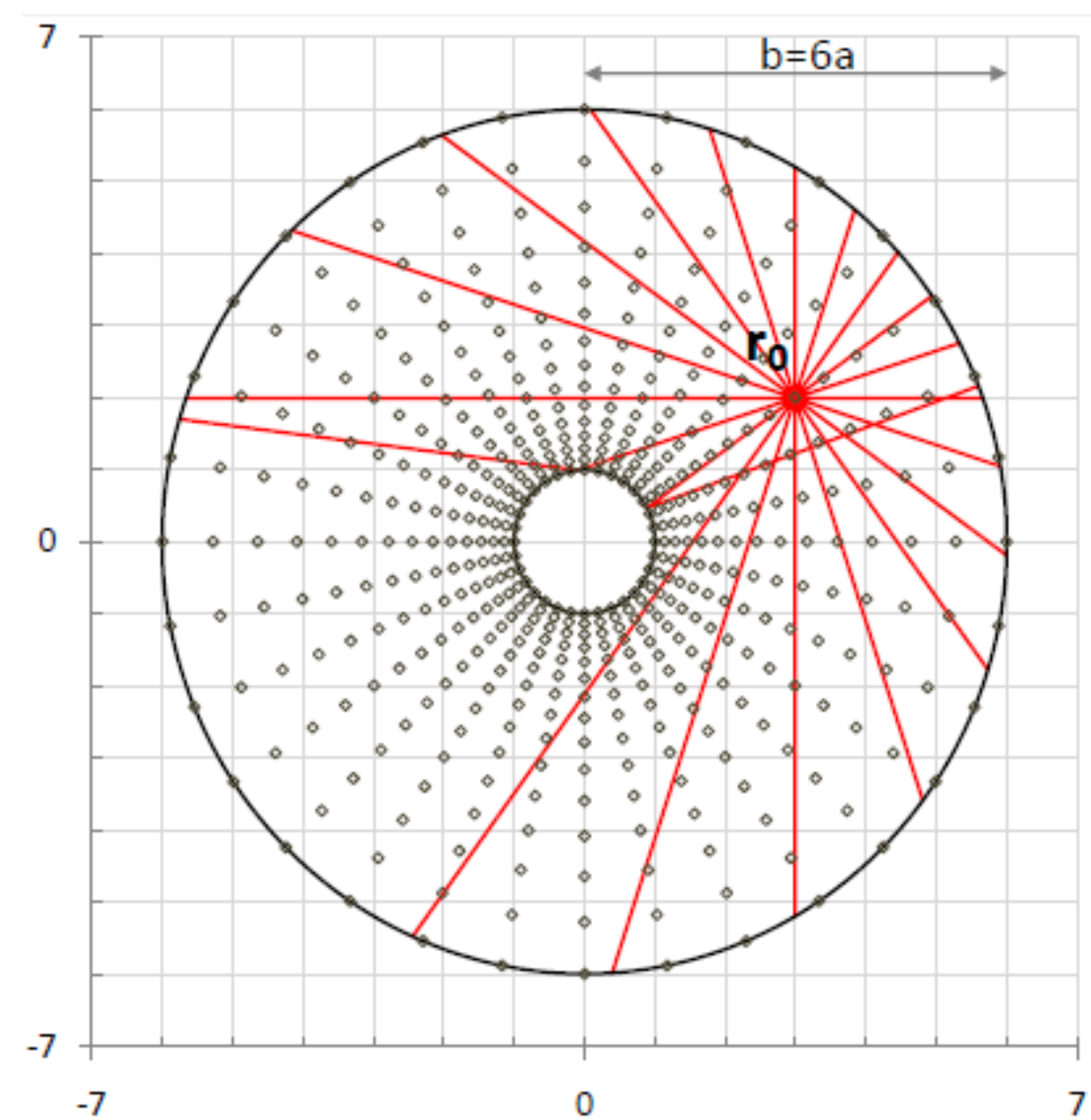
oscillating cylinder
in ^{3}He–^{4}He mixture

$$\delta\epsilon_{\hat{\boldsymbol{p}}}(\boldsymbol{r},t) = (1+\alpha)\delta\mu_{\mathrm{B}}(\boldsymbol{r},t) + Dp_{\mathrm{F}}\hat{\boldsymbol{p}}\cdot\boldsymbol{v}_s(\boldsymbol{r},t) + \sum_{l=0}^{\infty} F_l\langle P_l(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}')\phi_{\hat{\boldsymbol{p}}'}(\boldsymbol{r},t)\rangle_{\hat{\boldsymbol{p}}'},$$

kinetic equation

$$\frac{\partial\phi_{\hat{\boldsymbol{p}}}}{\partial t} + v_{\mathrm{F}}\hat{\boldsymbol{p}}\cdot\boldsymbol{\nabla}(\phi_{\hat{\boldsymbol{p}}} + \delta\epsilon_{\hat{\boldsymbol{p}}}) = I$$

relaxation time approximation for I
boundary conditions
Laplace equation to calculate $\boldsymbol{v}_s$ and μ_B
momentum flux tensor to calculate forces



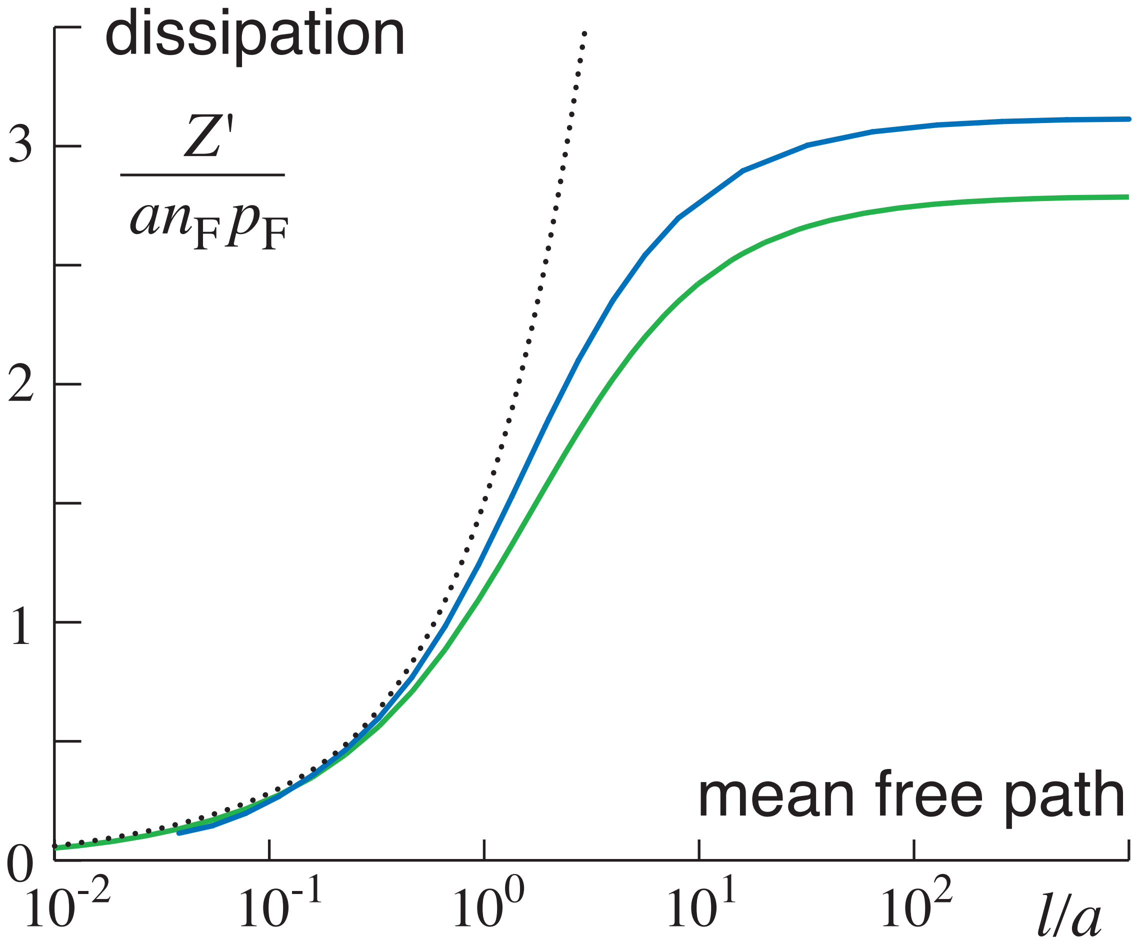
Results

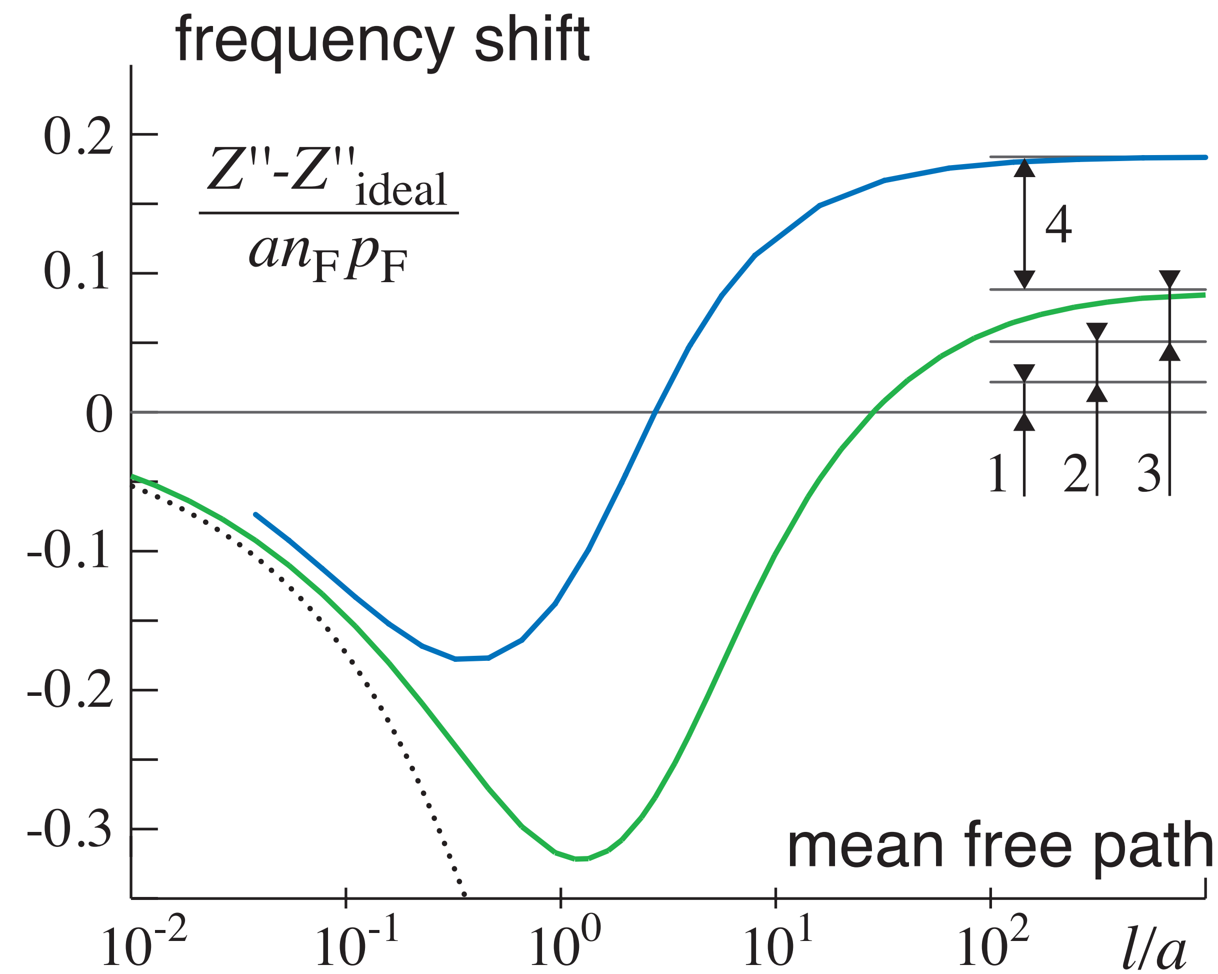
$\boldsymbol{F} = Z\boldsymbol{u}$

↑ force on the liquid

↑ cylinder velocity

$Z = Z' + iZ''$



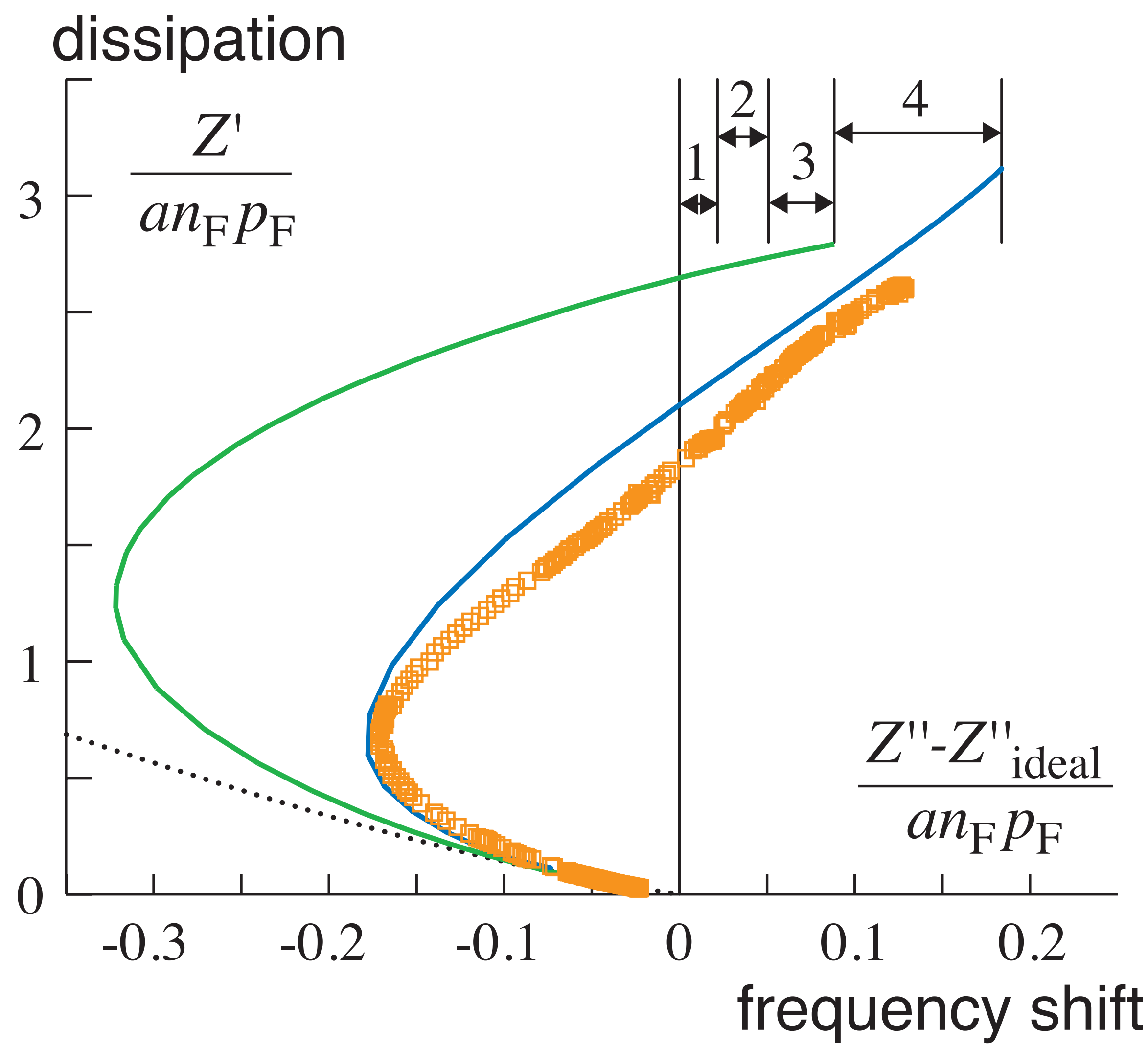


green: cylinder (radius a) in unlimited fluid
blue: cylinder in a slab of thickness $16a$
dotted: hydrodynamic approximation

1: decoupling of fermions
2: decoupling of bosons bound to quasiparticles
3: Landau force
4: force caused by quasiparticles reflected from slab walls

Vibrating wire experiment

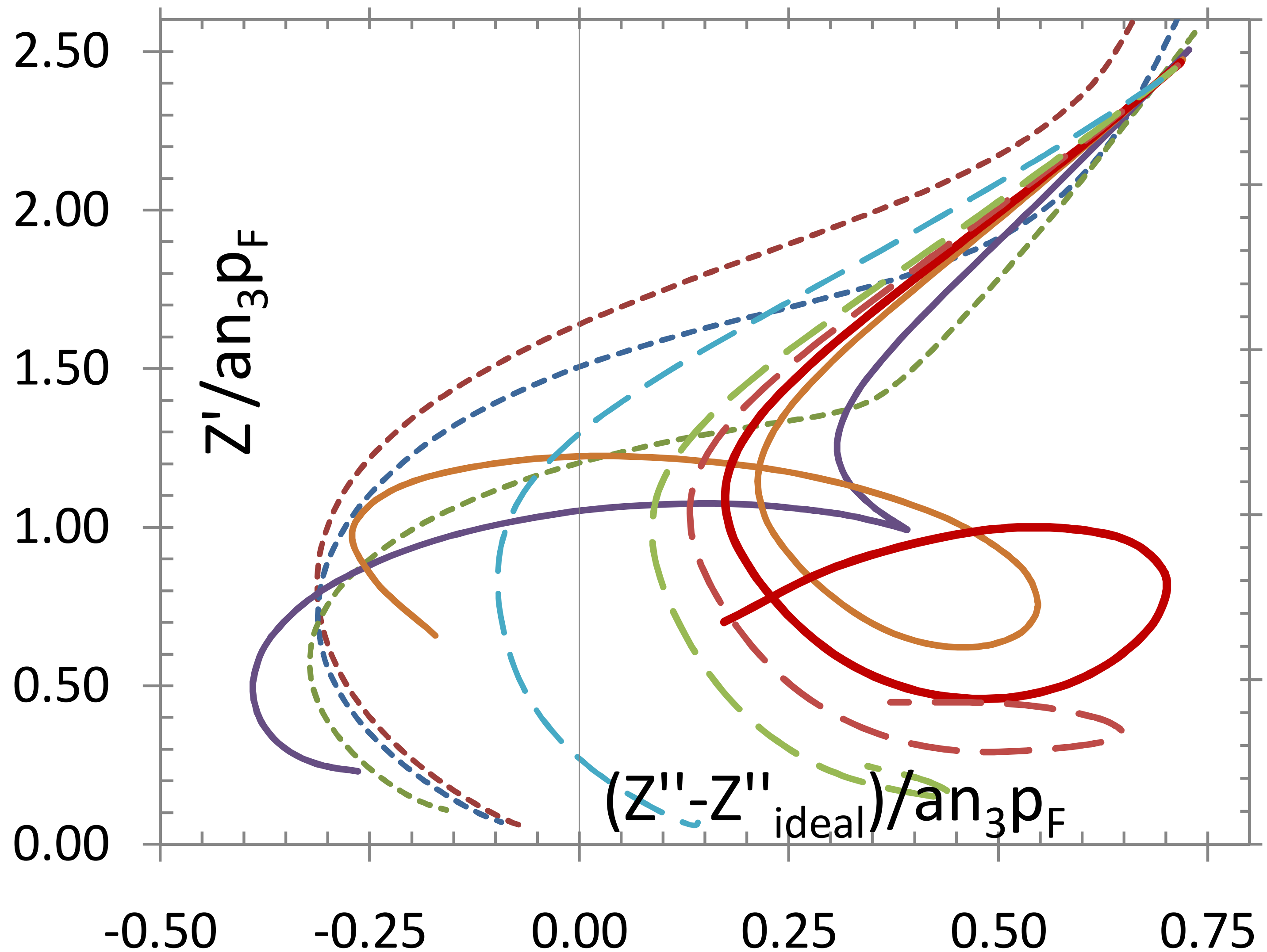
J. Martikainen, J. Tuoriniemi, T. Knuuttila, and G. Pickett (2002)



theory:
Fermi-liquid parameters taken from independent measurements,
diffusive bound conditions

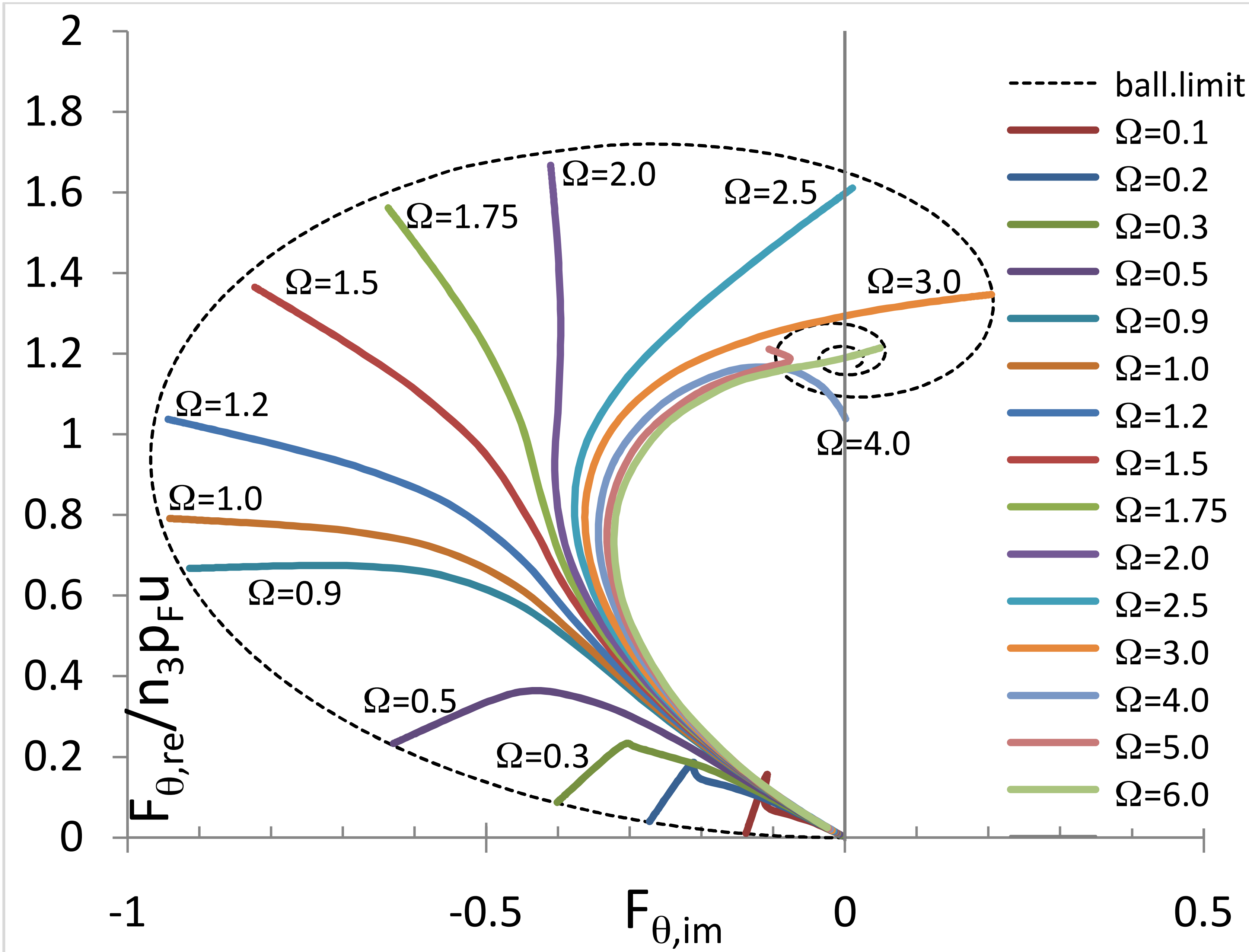
Further studies

damped second sound resonances at higher frequencies and in larger containers



Further studies 2

liquid inside a torsionally oscillating cylinder



$$\Omega = \frac{\omega R}{v_F}$$

The limit $\Omega \to \infty$ corresponds to a transverse oscillating plane, Bekarevich&Khalatnikov (1961), Flowers&Richardson (1978)

Summary

There is a force on a macroscopic object caused by the Fermi-liquid interactions. It can be interpreted as elasticity of the Fermi liquid.

The liquid force on a oscillating cylinder is calculated and compared with experiments in ^{3}He-^{4}He mixtures.

More on Poster 15P-A032 (today, not in program booklet)

Further reading: PRL 106, 055301 (2011); PRB 83, 245137 (2011); PRB 83, 224521 (2011)