Topological quantum matter: From superfluid ^{3}He to modern materials
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The Difference between Semiconductor and Superconductor

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It is my great pleasure to give a talk in this symposium in honor of Matti Krusius 80 years birthday. His experiments in superfluid ^{3}He, my collaboration with his group and his guidance has been essential part of my scientific life since 1980's.

Through collaboration with Matti I have learned many aspects of the normal and superfluid ^{3}He liquid. The topic of this talk is one more example how ^{3}He can be useful for something else. I learned the topic through study of critical velocity in superfluid ^{3}He. After discussing the title topic, I will return to how this can be useful for understanding some aspects of the critical velocity in superfluid Fermi liquid.

The title of this talk is the difference between semiconductor and superconductor. This is something we have learned already as physics students, and we all can list a number of differences between semiconducting and superconducting materials. They are so different that it may even sound strange to compare them.

My question is what is the most decisive single difference between semiconductors and superconductors?

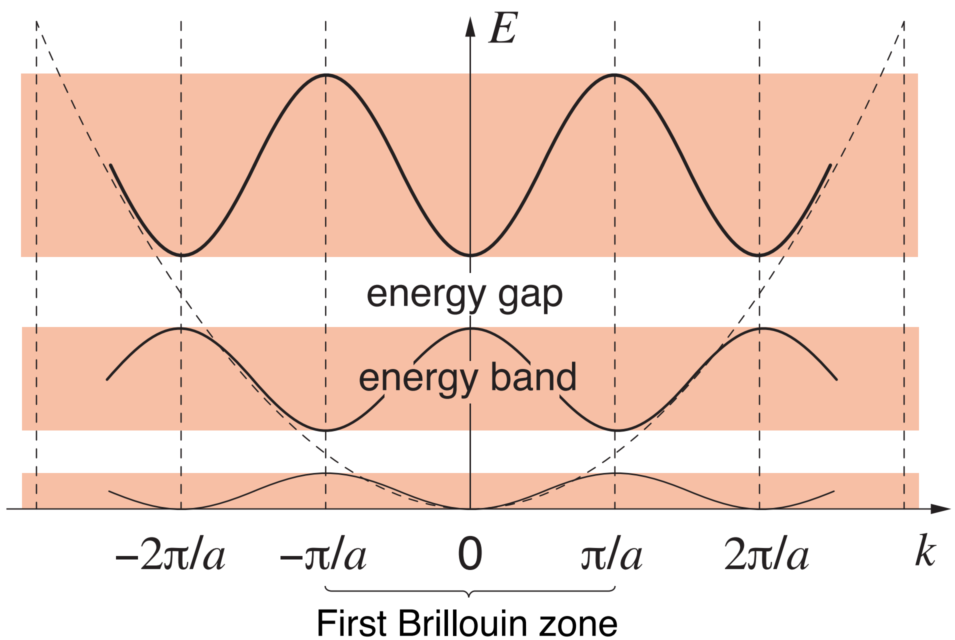
Before going to what I believe is the answer, let us briefly recall basics solid state physics (see books by Kittel, Ashcroft and Mermin, Hook and Hall, Simon etc.)

An important quantity in electronic structure of materials is the dependence of electron energy E on the wave vector $\boldsymbol{k}$: $E(\boldsymbol{k})$.

In free electron model $E(\boldsymbol{k})$ takes the simple form

$$E = \frac{\hbar^2 k^2}{2m}. \tag{1}$$

Taking into account the potential caused by the crystal lattice, the dispersion relation becomes more complicated: energy gaps open at $k = \pi n/a$, where a is the lattice constant and n is an integer.



For simplicity we take $\boldsymbol{k}$ parallel to one crystal axis. In true crystal the other directions have to be considered as well.

Notice that crystal wave vector k can be limited to the first Brillouin zone (which is called reduced-zone scheme). Then the levels at the boundary of the zone look like being discontinuous. This can be avoided using the extended-zone scheme (as in the figure above). There one has to remember that the energy levels shifted by $k \to k + 2\pi n/a$ are not independent, but represent the same physical electron level. That is, the occupation $f(E, k) \equiv f(E, k + 2\pi n/a)$.

In a crystal it is natural to describe the electron states in a frame of reference, where the crystal is at rest. We use that frame here, and where important to stress it, we call it "crystal frame"

The energy levels are filled with electrons starting from the bottom and following the Pauli principle. Solids can be divided in three groups based on the filling of the energy bands.

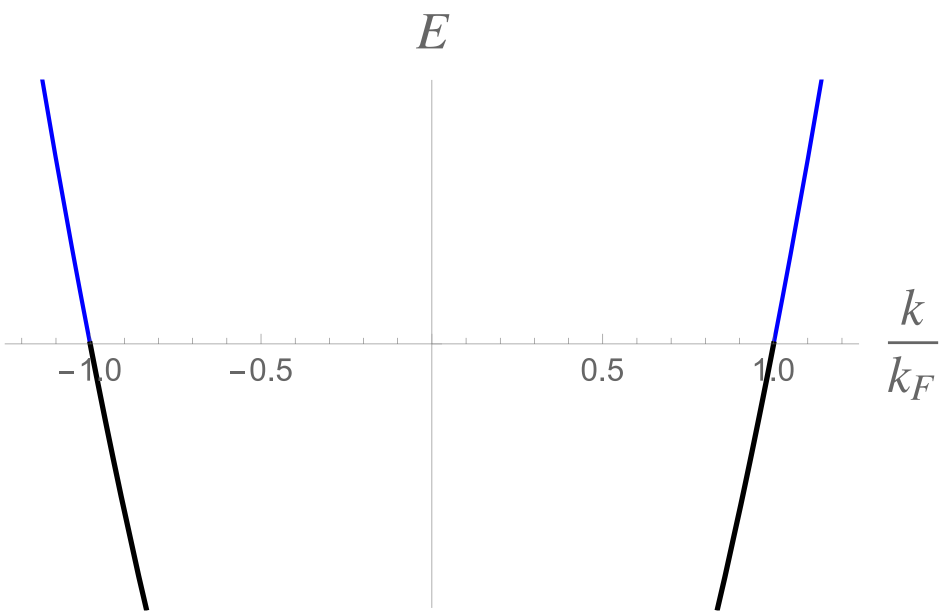
- Metal: there is one partially filled band (or more).

- Insulator: all bands are either filled or empty. The highest filled band is separated from the lowest empty band by an energy gap of several electron volts.

- Semiconductor: like insulator, but the band gap is smaller.

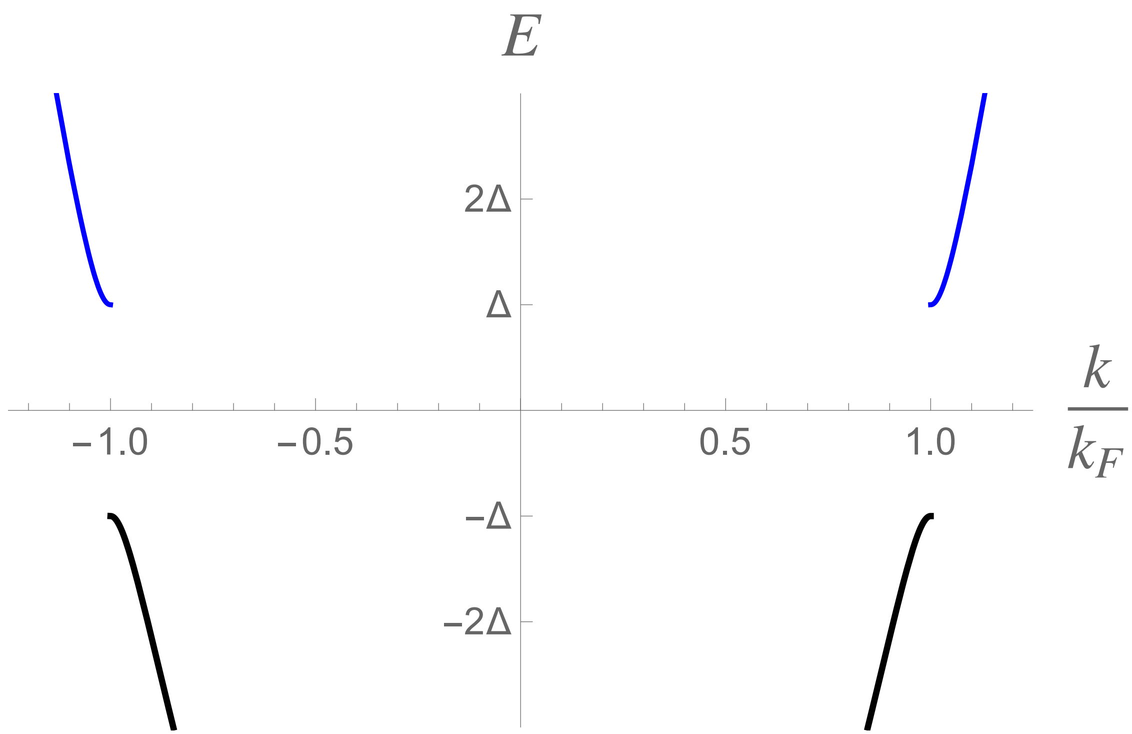
This is as much as we need to know about semiconductors at the moment. Let us turn to superconductivity.

As the staring point we consider a metal. The figure depicts the band dispersion near the Fermi surface.

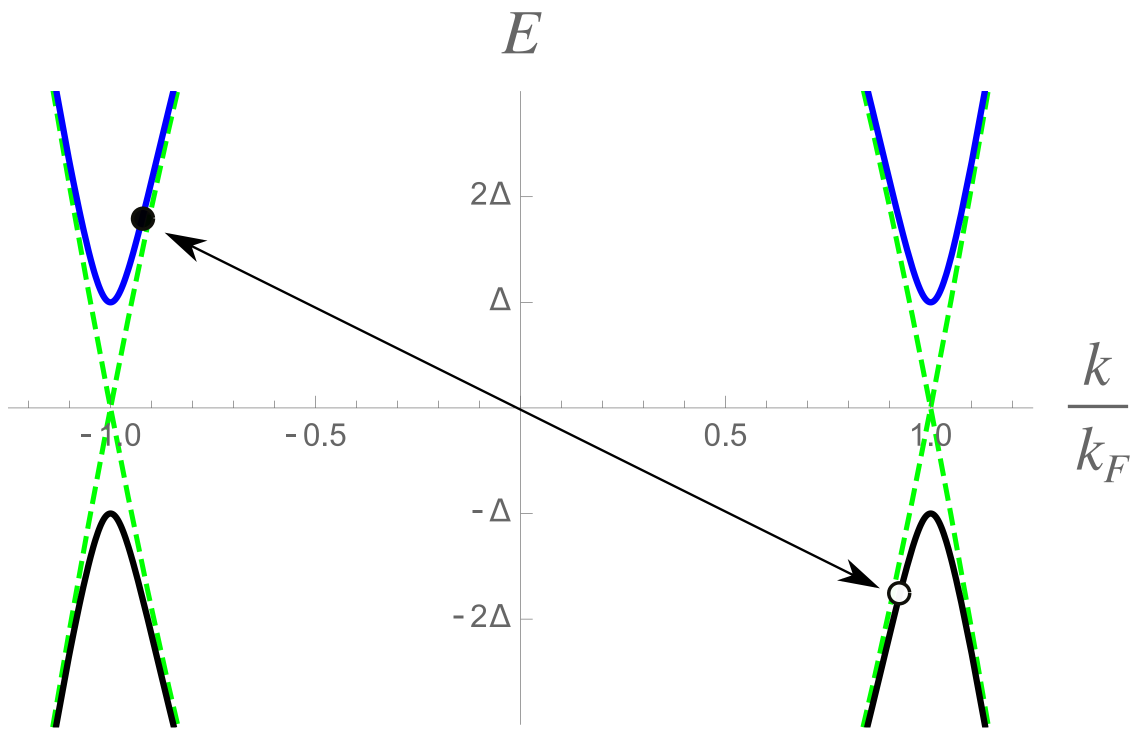


Here black lines denote occupied levels and the blue lines unoccupied levels at zero temperature ($T = 0$). It is also convenient to choose the Fermi level as the zero of energy, $E = 0$.

In the superconducting state an energy gap opens at the Fermi surface.



This picture looks discontinuous at $k = \pm k_F$. Therefore it is useful to introduce an extended scheme.

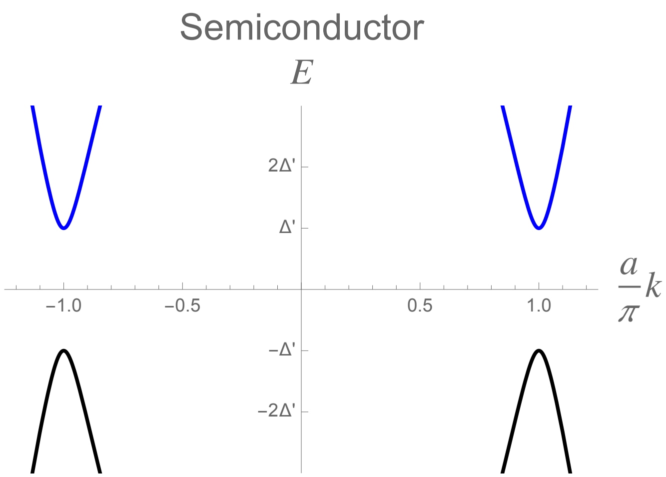


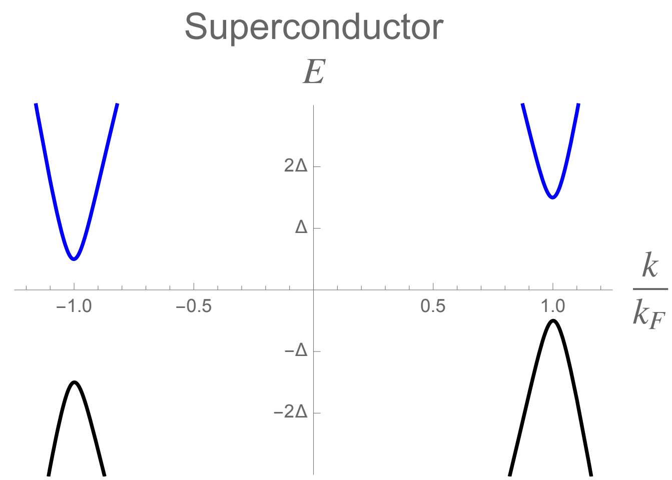
The figure shows extended scheme in the superconducting state (continuous lines) and in the normal state (dashed lines). The extended scheme has double representation of each physical level. The rule for the occupations is $f(E,k) \equiv 1 - f(-E,-k)$, as illustrated by the arrow in the figure. This means that a hole (missing electron) at a negative energy level is equivalent to an electron at the corresponding positive energy level of opposite wave vector.

The extended scheme is useful because depending on the need, one can select different “reduced schemes”. Selecting only positive energies gives the standard BCS excitation spectrum. Selecting the part shown in second last picture, one gets simple correspondence with the free particle dispersion (1). This is also used in connection of tunneling, where it is called *semiconductor model* (see Tinkham’s book). More advantage is to come.

We have seen that both semiconductors and superconductors have an energy gap that separates the filled states from the empty states at $T = 0$. What is the difference that allows dissipationless current only for the latter?

My answer is that in semiconductors the gap is fixed in the crystal frame, but in superconductors it can be inclined.

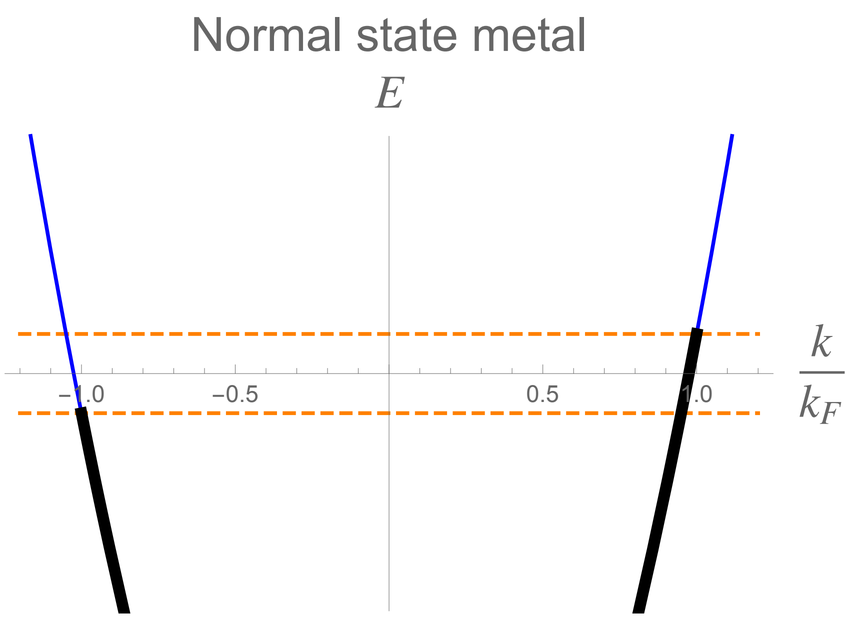




In semiconductors the gap arises from the crystal lattice and thus is balanced in the crystal frame.

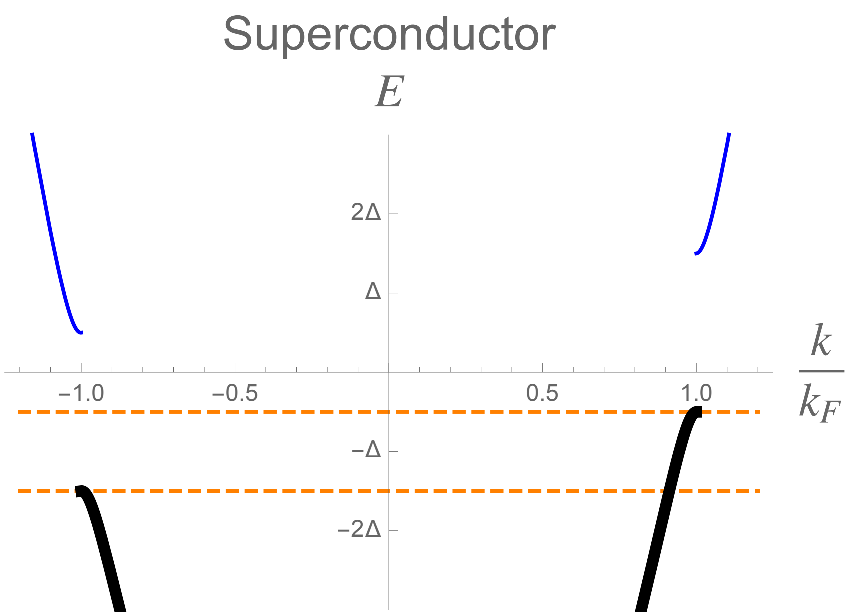
In superconductors, the gap arises from interaction of electrons. Although the interaction is mediated by the lattice, the gap is not necessarily horizontal in the crystal frame. The inclined gap is equivalent that the rest frame of the electron cloud is moving relative to the crystal frame: $E(\boldsymbol{k}) = E_0(\boldsymbol{k}) + \hbar\boldsymbol{k}\cdot\boldsymbol{v} + O(v^2)$, where $\boldsymbol{v}$ is the velocity.

Such relative motion does not lead to dissipation. To see that, let us recall the reason for resistance in normal-state metals. The electric field drives the electron distribution to one side. The electrons are scattered by impurities to the empty states of the same energy.



The electrons between the horizontal dashed lines are scattered to all directions having empty states. The 1D figure shows only the opposite direction.

In superconductors the scattering is ineffective because there are no levels of the same energy because of the energy gap.



This leads us to interprete that the supercurrent arises from the fact that there are more filled fermionic levels at $k \approx +k_F$ than at $k \approx -k_F$: the black line is longer on the right than on the left hand side of the figure.

In fact, this interpretation has firm theoretical justification. The following formula can be derived based on so-called "quasiclassical theory". (A better name would be "Fermi-liquid theory of superconductivity", Serene&Rainer 1983)

$$\boldsymbol{j} = ev_F N(0) \int \frac{d\Omega_k}{4\pi} \hat{\boldsymbol{k}} \int_{-E_c}^{E_c} d\epsilon \frac{|\tilde{\epsilon}|}{\sqrt{\tilde{\epsilon}^2 - |\Delta|^2}} \theta(\tilde{\epsilon}^2 - |\Delta|^2)(\phi_{B1} + \phi_{B2}), \quad (2)$$

Here e is the electron charge, v_F the Fermi velocity, $2N(0) = m^* k_F / \pi^2 \hbar^2$ is the quasiparticle density of states at the Fermi surface in the normal state. $\int d\Omega_k$ denotes integration over the directions $\hat{\boldsymbol{k}}$, E_c is a high energy cutoff and $\theta(x)$ is the Heaviside step function. ϕ_{B1} and ϕ_{B2} are quasiparticle distributions for particle and hole type excitations, respectively. They are symmetrized with respect to energy and in equilibrium they are $\phi_{B1} = \phi_{B2} = -\frac{1}{2} \tanh(\epsilon/2T) = f(\epsilon) - 1/2$.

Most importantly, $\tilde{\epsilon} = \epsilon - a$. Here $a(\boldsymbol{k}) \approx \boldsymbol{\alpha} \cdot \hat{\boldsymbol{k}}$ gives the inclination of the gap. For $\alpha = 0$, the gap is horizontal and with equilibrium distributions the integrations in (2) give vanishing current. For $\alpha \neq 0$ a nonvanishing supercurrent is found. The figure above is just the graphical representation of the formula (2).

A summary thus far:

- The principal difference of superconductor to semiconductor is that the gap can be inclined in the crystal frame.

- The supercurrent can be interpreted to arise from filled fermion levels at energies below the inclined gap.

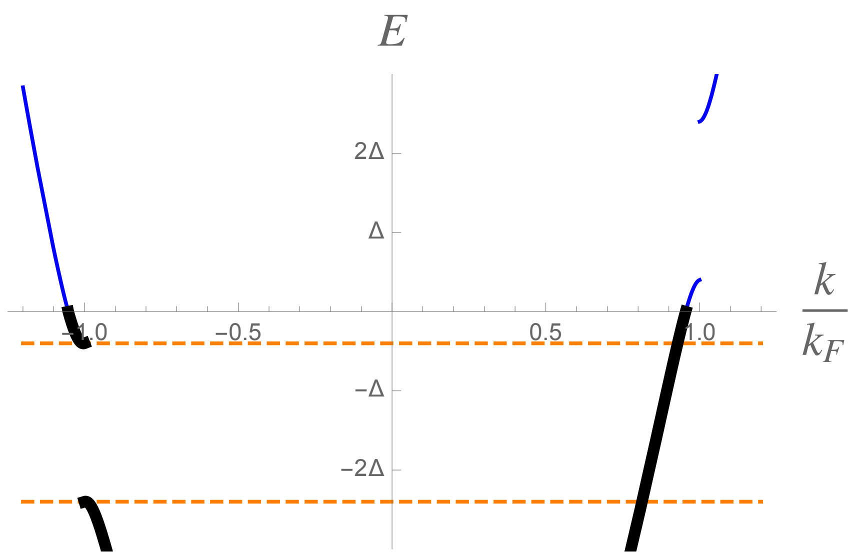
I showed that these results are implicit in the literature from the last century. I have no doubt that some reserchers have known this for long. However, I have some indications that this is not widely known:

- In spite of 40 years of research work on superfluidity/superconductivity I realized this only in recent years

- Edouard Sonin wrote paper on SNS junction [Phys. Rev. B **104**, 094517 (2021)] which is in disagreement with well established results from 1970's. I wrote a comment (arXiv:2112.07378) but using concepts presented above. That was a disaster: one referee partly accepted my view but the other did not understand it at all. The manuscript is now in appeal stage.

- I have not encountered material presenting the concepts explicitly. Especially I could imagine that the results could be used in teaching condensed matter physics, but I have not seen any teaching material using them. Maybe you know?

The picture above we can used to illustrate several effects. Consider the case that the inclination of the dispersion is so large that some states below the gap are lifted to positive energies. At $T = 0$ the equilibrium occupations are given by the black lines in the figure.

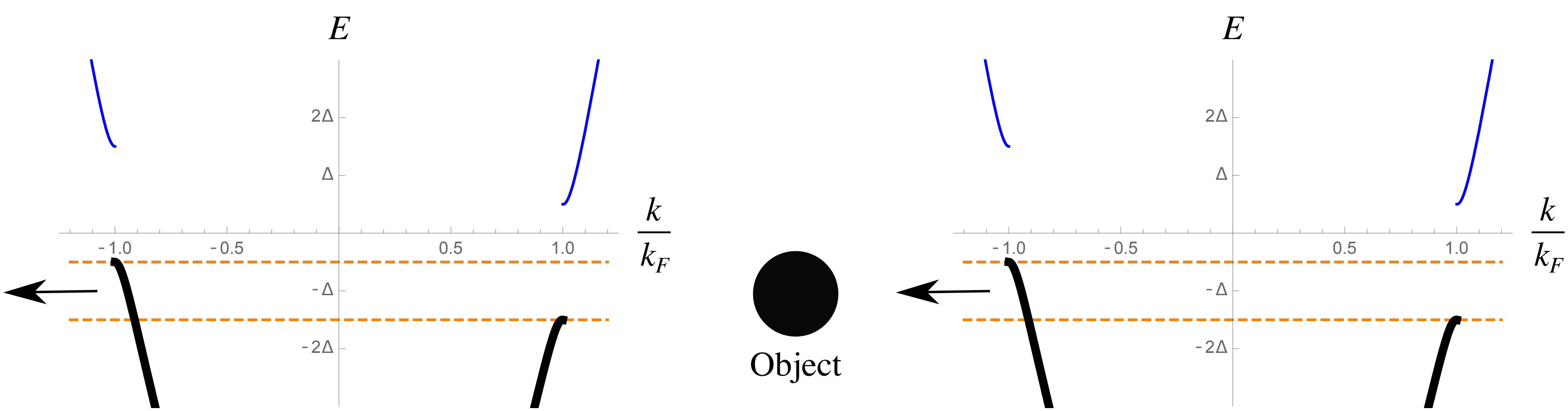


We see that the filled states are not balanced, so there can be supercurrent. This applies, in particular, to anisotropic superconductors which have nodes in the gap. Although the gap vanishes in particular $\hat{\boldsymbol{k}}$ directions , there can supercurrent as long as the gap persists in some directions.

Drag on an object in superfluid Fermi liquid in $T \to 0$ limit

This problem is related to the one above, but there are important differences. Instead of the crystal we now have a compact object and the fermion fluid is not inside but outside.

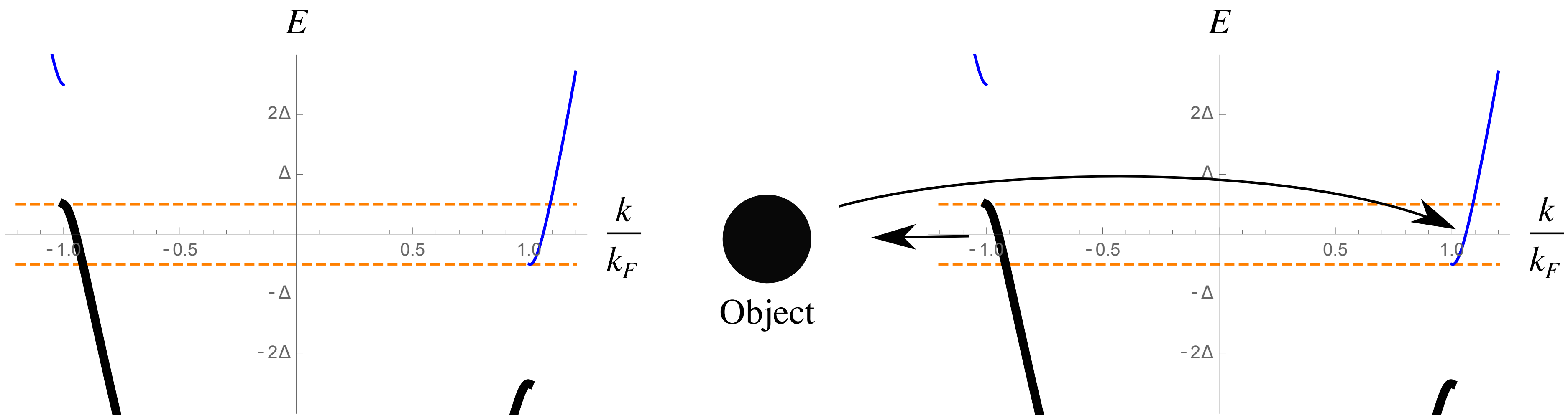
We study the problem in the rest frame of the object (object frame), and the fluid is moving past the object at velocity v (to the left). The quasiparticle dispersion in the fluid is illustrated on both sides of the object in the figure.



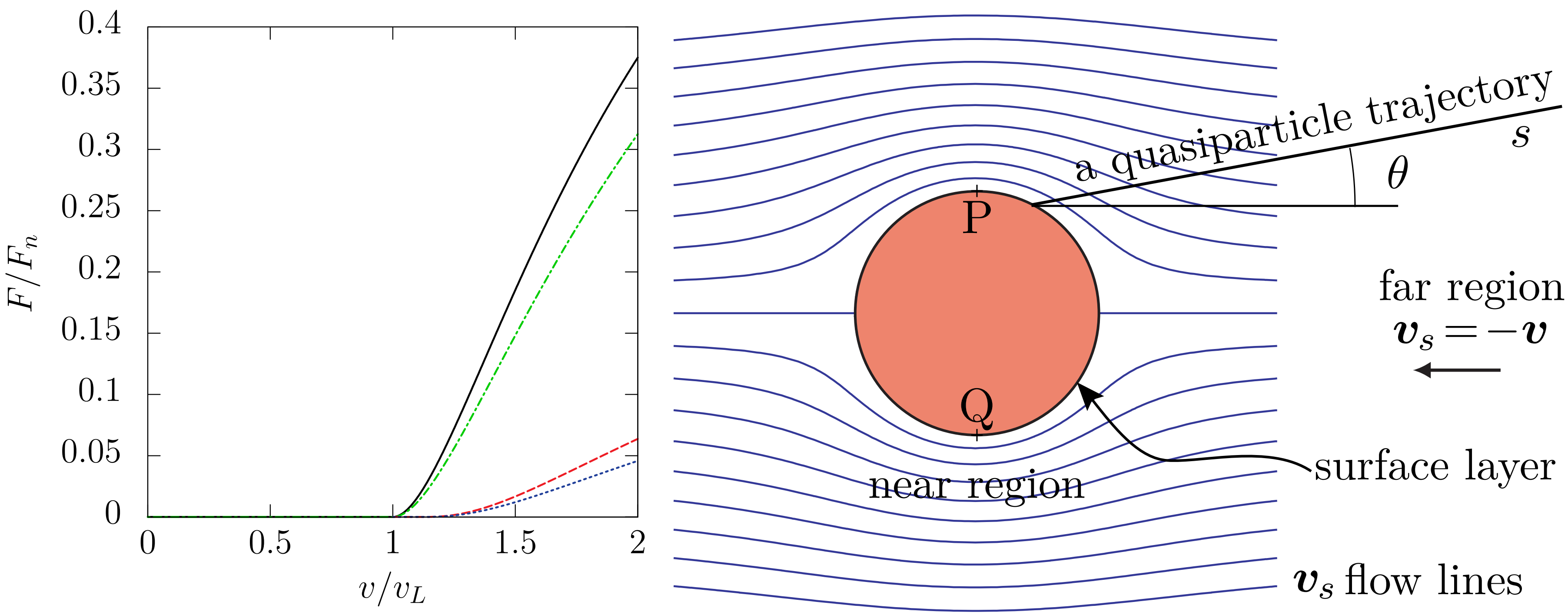
The fermions hit the object more from the right (denoted by arrow). In the case of the figure above, all empty states are at higher energy than the filled ones. Thus the object cannot induce scattering and the drag force vanishes. This is the case at velocities below the Landau velocity, $v < v_L$,

$$v_L = \frac{\Delta}{p_F} \tag{3}$$

The quasiparticle dispersion in the fluid at supercritical velocity ($v > v_L$) is illustrated below.

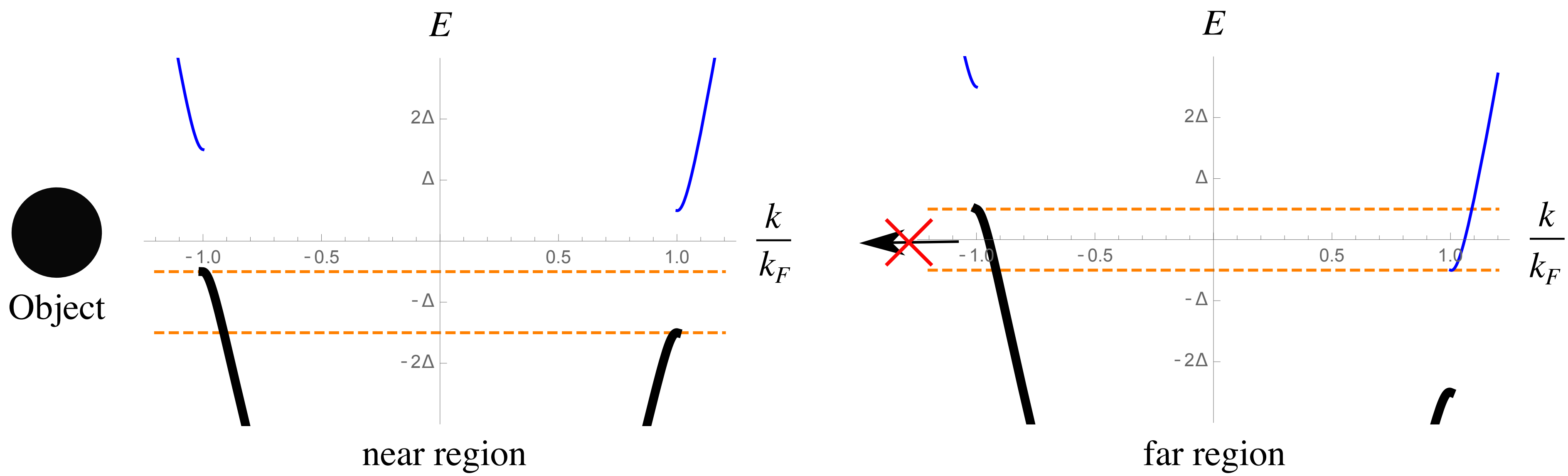


The quasiparticles in the indicated range (between dashed lines) hitting the object from the left are scattered to the empty states, which leads to drag force.



Left: the theoretical drag force on a small object (upper curves) and for a macroscopic object (lower curves). [Kuorelahti, Laine and T, Phys. Rev. B 98, 144512 (2018)]

Right: for a macroscopic object one has to take into account the effect of the flow field in the near region of the object



The figure illustrates that the modified fluid velocity in the near region can prevent quasiparticles from colliding with a macroscopic object and thus lead to reduced drag. We have studied a few models for the flow field and found the drag indeed is reduced. In spite of lower drag for macroscopic object, the models we have used are not able to explain such a low drag as observed in Lancaster [D. I. Bradley, S. N. Fisher, A. M. Guénault, R. P. Haley, C. R. Lawson, G. R. Pickett, R. Schanen, M. Skyba, V. Tsepelin, and D. E. Zmeev, *Breaking the Superfluid Speed Limit in a Fermionic Condensate*, Nat. Phys. **12**, 1017 (2016).]

Summary

- The principal difference of superconductor to semiconductor is that the gap can be inclined in the crystal frame.

- The supercurrent can be interpreted to arise from filled fermion levels at energies below the inclined gap.

- These concepts are useful in understanding critical supercurrents and the drag on moving objects in Fermi superfluids.